Non-linear Unmixing of Hyperspectral Images: Myth or Reality?

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ANR HYPANEMA, Projet ANR-12-BS03-003

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3ème Colloque de la SFPT-GH
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Outline

Outline

Linear Mixing Model

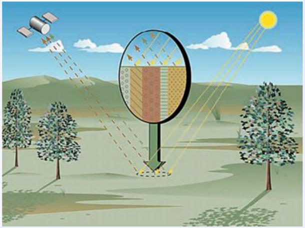


Figure : Linear mixing model[1]

- Single path of the photons
- Observations: sums of the individual contributions of the endmembers

[1] N. Keshava and J. F. Mustard, "Spectral Unmixing," IEEE Signal Processing Magazine, vol. 19, no 1, pp. 44-57, 2002.

Linear Mixing Model

$$\mathbf{y} = \sum_{r=1}^{R} a_r \mathbf{m}_r + \mathbf{n} = \mathbf{M}\mathbf{a} + \mathbf{n}$$

$$\mathbf{a} = [a_1, \ldots, a_R]^T, \mathbf{M} = [\mathbf{m}_1, ..., \mathbf{m}_R]$$

- $\mathbf{y}$: observation vector of size L
- $\mathbf{m}_r$: rth endmember spectrum
- a_r: rth endmember abundance
- $\mathbf{n}$: noise sequence such that $\mathbf{n} \sim \mathcal{N}\left(\mathbf{0}_L, \sigma^2 \mathbf{I}_L\right)$
- R: number of endmembers present in the scene

Linear Mixing Model

Physical constraints

$$\text{Positivity:} \quad a_r \geq 0, \forall r \in 1, ..., R, \quad \text{Sum-to-one:} \quad \sum_{r=1}^{R} a_r = 1$$

Remarks

- Realistic first order approximation
- No scattering effects
- No interaction between components

Outline

Nonlinear Mixing Models

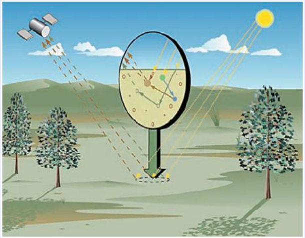


Figure : Nonlinear mixing model[1]

Why?

- Possible interactions between the different materials
- Intimate mixtures

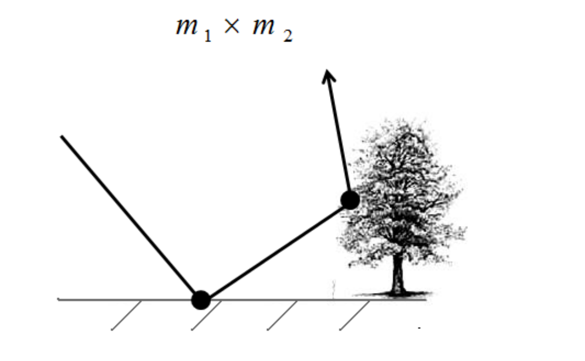
(1) N. Keshava and J. F. Mustard, "Spectral Unmixing," IEEE Signal Processing Magazine, vol. 19, no 1, pp. 44-57, 2002.

Bilinear Models: Fan (FM)

Definition

W. Fan, B. Hu, J. Miller, and M. Li, "Comparative study between a new nonlinear model and common linear model for analysing laboratory simulated forest hyperspectral data," *International Journal of Remote Sensing*, June 2009.

$$\boldsymbol{y} = \sum_{r=1}^{R} a_r \boldsymbol{m}_r + \sum_{i=1}^{R-1} \sum_{j=i+1}^{R} a_i a_i \boldsymbol{m}_i \odot \boldsymbol{m}_j + \boldsymbol{n}$$



Fan Model (FM)

Test images
Simulated images (in laboratory) corresponding to sunlit tree crowns, sunlit background and shadow

Estimation strategy
Linearization using Taylor expansion + Least Squares

Bilinear Models: Nascimento

Definition

J. M. P. Nascimento and J. M. Bioucas-Dias, "Nonlinear mixture model for hyperspectral unmixing," in *Proc. SPIE*, Sept. 2009.

$$\boldsymbol{y} = \sum_{r=1}^{R} a_r \boldsymbol{m}_r + \sum_{r=1}^{R-1} \sum_{j=r+1}^{R} \beta_{r,j} \boldsymbol{m}_r \odot \boldsymbol{m}_j + \boldsymbol{n}$$

Constraints

Positivity

$$a_r \geq 0, \quad \beta_{r,j} \geq 0, \quad \forall r \in 1, ..., R \quad \forall j \in r+1, \ldots R$$

Sum-to-one

$$\sum_{r=1}^{R} a_r + \sum_{i=1}^{R-1} \sum_{j=i+1}^{R} \beta_{i,j} = 1$$

Nascimento Model

Test images

- Simulated images corresponding to trees, grass and shadow
- Real images from Landgrebe's book

Estimation strategy

Introduction of virtual endmembers + Fully constrained Least Squares

Bilinear Models: Generalized Bilinear Model (GBM)

Definition

A. Halimi, Y. Altmann, N. Dobigeon and J.-Y. Tourneret, "Nonlinear unmixing of hyperspectral images using a generalized bilinear model," *IEEE Trans. Geosci. Remote Sens.*, Nov. 2011.

$$\boldsymbol{y} = \sum_{r=1}^{R} a_r \boldsymbol{m}_r + \sum_{i=1}^{R-1} \sum_{j=i+1}^{R} a_i a_j \gamma_{i,j} \boldsymbol{m}_i \odot \boldsymbol{m}_j + \boldsymbol{n}$$

- $\boldsymbol{\gamma} = [\gamma_{1,2}, \gamma_{1,3}, \ldots, \gamma_{R-1,R}]^T$ vector allowing the interactions (nonlinearities) between the endmembers to be quantified

$$0 \leq \gamma_{i,j} \leq 1, \forall i \in 1, \cdots, R-1, \forall j \in 1, \cdots, R,$$

- $\mathbf{a} = [a_1, \ldots, a_R]^T$ abundance vector satisfying

$$a_r \geq 0, r = 1, \cdots, R \quad \text{and} \quad \sum_{r=1}^{R} a_r = 1.$$

Generalized Bilinear Model (GBM)

$$\boldsymbol{y} = \sum_{k=1}^{R} a_r \boldsymbol{m}_k + \sum_{i=1}^{R-1} \sum_{j=i+1}^{R} a_i a_j \gamma_{i,j} \boldsymbol{m}_i \boldsymbol{m}_j + \boldsymbol{n}$$

Properties

The GBM generalizes 2 mixing models

- $\boldsymbol{\gamma} = [0, 0, \ldots, 0]^T \rightarrow$ LMM
- $\boldsymbol{\gamma} = [1, 1, \ldots, 1]^T \rightarrow$ bilinear FM

The vector $\boldsymbol{\gamma}$ allows the level of interactions (nonlinearities) to be quantified for each pixel.

Generalized Bilinear Model (GBM)

Test images

- Simulated data generated according to linear and nonlinear models
- Real AVIRIS images: Moffet field + Cuprite

Estimation strategy

- Bayesian algorithm coupled with MCMC methods
- Optimization algorithms: linearization or gradient descent method

New Interesting Results

Physical model (radiative transfer theory)

I. Meganem, P. Déliot, X. Briottet, Y. Deville and S. Hosseini, "Linear-Quadratic Mixing Model for Reflectances in Urban Environments," *IEEE Trans. Geoscience and Remote Sensing*, vol. 52, no 1, pp. 544-558, Jan. 2014

$$\boldsymbol{y} = \sum_{r=1}^{R} a_r \boldsymbol{m}_r + \sum_{r=1}^{R} \sum_{j=r}^{R} \beta_{r,j} \boldsymbol{m}_r \odot \boldsymbol{m}_j + \boldsymbol{n}$$

Constraints

Positivity

$$a_r \geq 0, \quad \beta_{r,j} \geq 0, \quad \forall r \in 1, ..., R \quad \forall j \in r, \ldots R$$

Sum-to-one

$$\sum_{r=1}^{R} a_r = 1$$

Simulated urban test images

Bilinear Models: Endmember Estimation

Nearest-neighbor graph on the data

R. Heylen, D. Burazerovic and P. Scheunders, "Non-linear spectral unmixing by geodesic simplex volume maximization," *IEEE J. Sel. Topics Sig. Process.*, June 2011.

Matrix factorization

- N. Yokoya, J. Chanussot and A. Iwasaki, "Nonlinear unmixing of hyperspectral data using semi-nonnegative matrix factorization," *IEEE Trans. Geoscience and Remote Sensing*, Feb. 2014.
- O. Eches and M. Guillaume, "A bilinear-bilinear nonnegative matrix factorization method for hyperspectral unlmixing," *IEEE Geoscience and Remote Sensing Lett.*, Apr. 2014.

Bayesian inference

Y. Altmann, N. Dobigeon and J.-Y. Tourneret, "Unmixing bilinear models using a Bayesian method and Hamiltoninan MCMCs," in preparation.

Post Nonlinear Mixing Model (PPNMM)

Definition

Y. Altmann, A. Halimi, N. Dobigeon and J.-Y. Tourneret "Supervised nonlinear spectral unmixing using a polynomial post nonlinear model for hyperspectral imagery," *IEEE Trans. Image Process.*, June 2012.

$$\mathbf{y} = g\left(\sum_{r=1}^{R} a_r \mathbf{m}_r\right) + \mathbf{n}$$

where g is an invertible nonlinear application from $\mathbb{R}^L$ to $\mathbb{R}^L$.

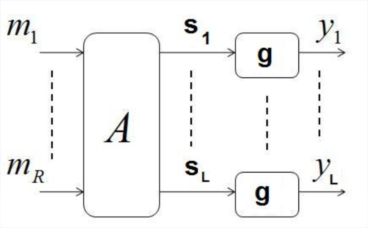
Constraints

$$a_r \geq 0, \quad \forall r \in 1, ..., R, \text{ and } \sum_{r=1}^{R} a_r = 1$$

Polynomial Post Nonlinear Mixing Model

$$
\begin{aligned}
g: \quad [0,1]^L \quad &\rightarrow \mathbb{R}^L \\
\mathbf{s} \quad &\mapsto \left[g(s_1) = s_1 + b s_1^2, \ldots, g(s_L) = s_L + b s_L^2\right]^T
\end{aligned}
$$

where $\mathbf{s} = [s_1, \ldots, s_L]^T$



Resulting Model

$$
\mathbf{y} \quad = \quad \mathbf{Ma} + b\,(\mathbf{Ma}) \odot (\mathbf{Ma}) + \mathbf{n}
$$

Polynomial Post Nonlinear Mixing Model: Abundance Estimation

Bayesian method

- Definition of prior distributions satisfying the constraints
- Derivation of the posterior distribution
- Simulation of samples using a Markov Chain Monte Carlo (MCMC) method

Optimization methods

- Linearization using Taylor-series expansions
- Steepest descent method

Polynomial Post Nonlinear Mixing Model: Endmember Estimation

Nearest-neighbor graph on the data

R. Heylen, D. Burazerovic and P. Scheunders, “Non-linear spectral unmixing by geodesic simplex volume maximization,” *IEEE J. Sel. Topics Sig. Process.*, June 2011.

Blind Non-linear Source Separation

- A. Taleb and C. Jutten, “Source separation in post-nonlinear mixtures,” *IEEE Trans. Signal Processing*, 1999.
- A. Ziehe, M. Kawanabe, S. Harmeling and K.-R. Müller, “Blind separation of post-nonlinear mixtures using linearizing transformations and temporal decorrelation,” *Journal of Machine Learning Research*, 2003.

Bayesian inference

Y. Altmann, N. Dobigeon and J.-Y. Tourneret,”Unsupervised Post-Nonlinear Unmixing of Hyperspectral Images Using a Hamiltonian Monte Carlo Algorithm”, to appear in IEEE Trans. on Image Process., 2014.

Spectral Unmixing Using Kernels

Definition

J. Chen, C. Richard and P. Honeine, “Nonlinear unmixing of hyperspectral data based on a linear-nonlinear fluctuation model,” *IEEE Trans. Signal Process.*, Jan. 2013.

$$\min \sum_{l=1}^{L} [y_l - \psi_{\boldsymbol{a}}(\boldsymbol{m}_{\lambda_l})]^2 + \mu ||\psi_{\boldsymbol{a}}||_H^2$$

where

- L is the number of spectral bands
- $\boldsymbol{m}_{\lambda_l} = (m_{l,1}, ..., m_{l,R})^T$ contains the endmember components in band $\#l$
- H is a given reproducing kernel hilbert space (RKHS)
- $\psi_{\boldsymbol{a}}$ is a known function defining nonlinear interactions between the endmembers
- μ is a positive parameter that controls the tradeoff between regularity of $\psi_{\boldsymbol{a}}$ and fitting

The Linear-Nonlinear Fluctuation Model

Definition

$$\psi_{\boldsymbol{a}}(\boldsymbol{m}_{\lambda_l}) = \boldsymbol{a}^T \boldsymbol{m}_{\lambda_l} + \psi_{\mathrm{nlin}}(\boldsymbol{m}_{\lambda_l})$$

with the constraints

$$a_r \geq 0, \quad \forall r \in 1, ..., R, \text{ and } \sum_{r=1}^{R} a_r = 1$$

Inference

- Abundance estimation using the reproducing kernel machinery
- Endmember estimation using the theory of Gaussian processes
  Y. Altmann, N. Dobigeon, S. McLaughlin and J.-Y. Tourneret, "Nonlinear spectral unmixing of hyperspectral images using Gaussian processes," *IEEE Trans. Signal Process.*, May 2013.

Outline

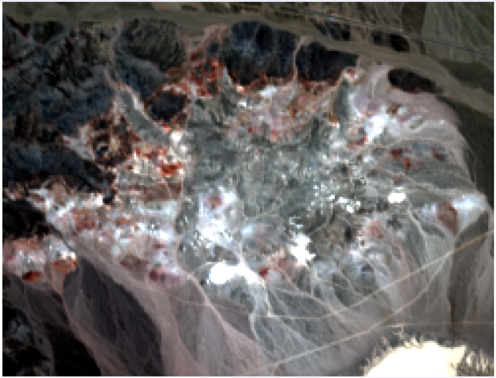


Figure : Cuprite Image

$$\begin{cases} H_0 & : \quad \text{y is distributed according to the LMM} \\ H_1 & : \quad \text{y is not distributed according to the LMM} \end{cases}$$

Detection Strategies

Post-nonlinear Mixing Model

$$\mathbf{y} = \mathbf{Ma} + b\,(\mathbf{Ma}) \odot (\mathbf{Ma}) + \mathbf{n}$$

Y. Altmann, N. Dobigeon and J.-Y. Tourneret, "Nonlinearity Detection in Hyperspectral Images Using a Polynomial Post-Nonlinear Mixing Model," *IEEE Trans. Image Process.*, April 2013.

$$\begin{cases} H_0 & : \quad b = 0 \\ H_1 & : \quad b \neq 0 \end{cases}$$

Detection Rule

$$\hat{T}^2 = \frac{\hat{b}^2}{\hat{s}_0^2} \underset{H_0}{\overset{H_1}{\gtrless}} \eta \quad \text{with} \quad \hat{s}_0^2 = \mathrm{CCRLB}(b = 0; \hat{\mathbf{a}}, \hat{\sigma}^2)$$

Synthetic Data

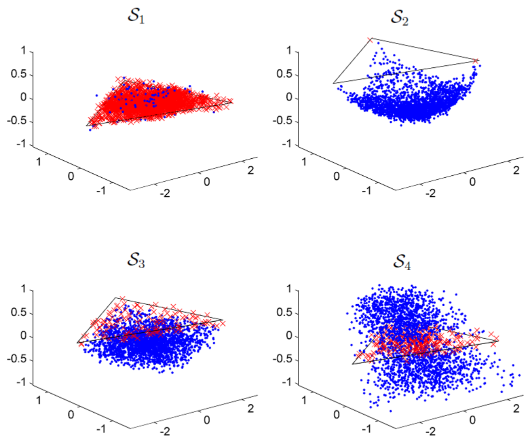
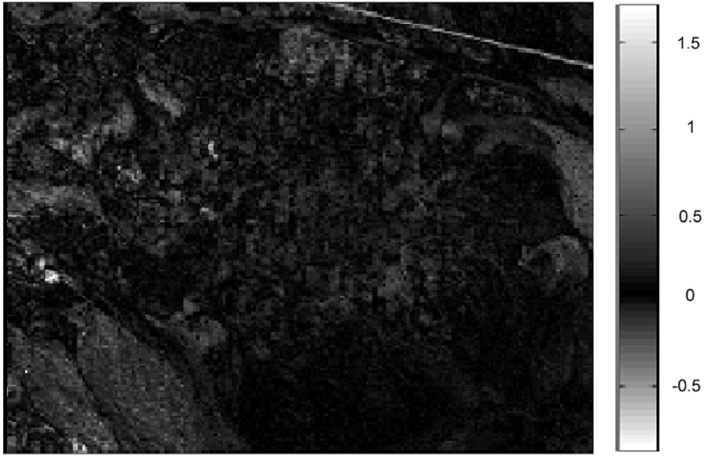
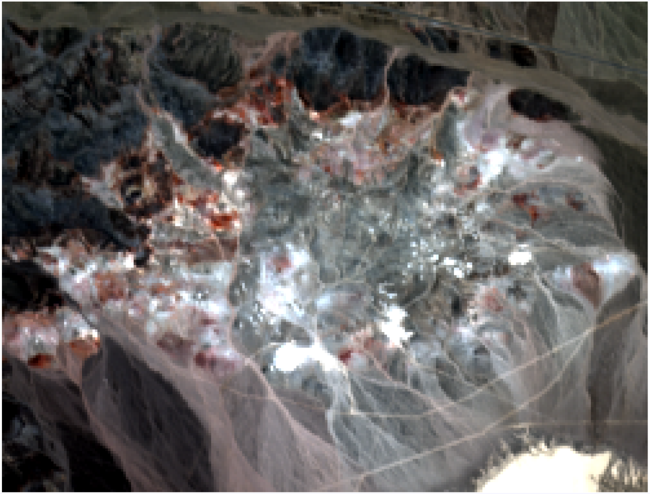


Figure : Pixels detected as linear (red crosses) and nonlinear (blue dots) for the four sub-images S_1 (LMM), S_2 (FM), S_3 (GBM) and S_4 (PPNMM).

Nonlinearities in Cuprite Image



Cuprite Image



Detection Strategies

Distance to the endmember simplex

$$\delta^2(\mathbf{y}) = \min_{\boldsymbol{z} \in \mathcal{H}} \|\mathbf{y} - \boldsymbol{z}\|^2$$

Y. Altmann, N. Dobigeon, J.-Y. Tourneret and J. C. Bermudez, “A robust test for nonlinear mixture detection in hyperspectral images,” in *Proc. ICASSP*, 2013.

Linear-Nonlinear Fluctuation Model

$$\mathbf{y}_l = \boldsymbol{a}^T \boldsymbol{m}_{\lambda_l} + \psi_{\text{nlin}}(\boldsymbol{m}_{\lambda_l}) + \text{noise}$$

Future work for C. Richard?

Outline

Conclusions

Non-linear Mixing Models

- Various parametric and non-parametric models based on bilinear, post-nonlinear transformations or kernels
- Various estimation algorithms based on (constrained) least-squares, Bayesian or machine learning methods

Recent Reference

N. Dobigeon, J.-Y. Tourneret, C. Richard, J. C. M. Bermudez, S. McLaughlin and A. O. Hero, "Nonlinear unmixing of hyperspectral images: models and algorithms," *IEEE Signal Processing Magazine*, Jan. 2014.

Review papers

Special Issue "Signal and Image Processing in HS Remote Sensing"
(Editors: W.-K. Ma, J. M. Bioucas-Dias, J. Chanussot and P. Gader)

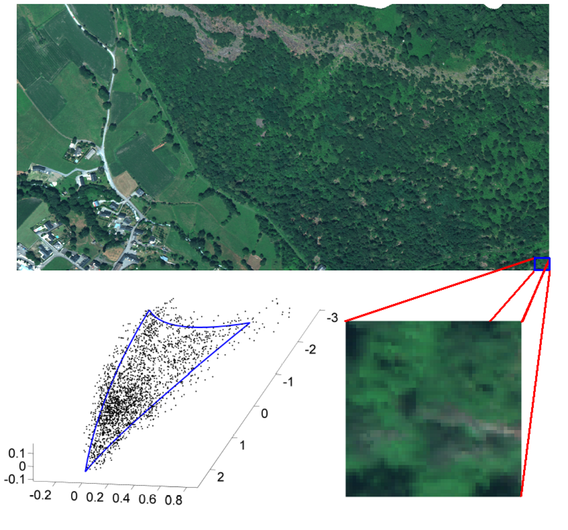
Review papers

Special Issue "Signal and Image Processing in HS Remote Sensing" (Editors: W.-K. Ma, J. M. Bioucas-Dias, J. Chanussot and P. Gader)

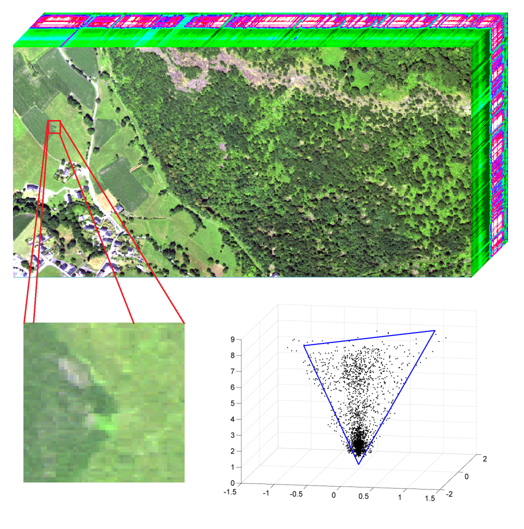
- W.-K Ma, J. Bioucas-Dias, T.-H. Chan, N. Gillis, P. Gader, A. Plaza, A. Ambikapathi and C.-Y. Chi, "*A signal processing perspective on hyperspectral unmixing.*," IEEE Signal Processing Magazine, Jan. 2014.

- N. Dobigeon, J.-Y. Tourneret, C. Richard, J. C. M. Bermudez, S. McLaughlin and A. O. Hero, "*Nonlinear unmixing of hyperspectral images: models and algorithms*," IEEE Signal Processing Magazine, Jan. 2014.

Myth or Reality? MADONNA - Scene 1



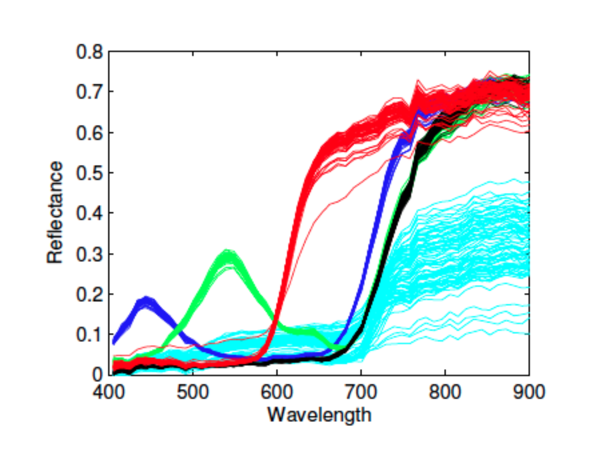
Myth or Reality? MADONNA - Scene 2



Outline

First Challenge: Endmember Variability

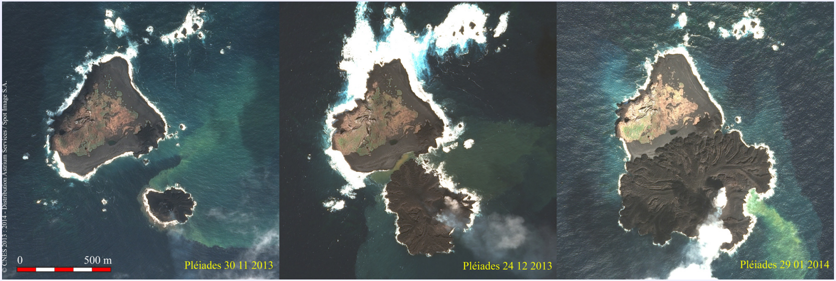
Endmember Spectra measured using a handheld ASD spectrometer. Blue: Blue Cloth, Green: Green Cloth, Red: Red Cloth, Black: Black Cloth



Thanks to Alina Zare from University of Missouri-Columbia for the picture

Second Challenge: Multi-Temporal Hyperspectral Imagery

Fusion of Snoopy and Nishino Japanese Islands - Pleiades Images



Thanks to the CNES of Toulouse for providing these Pleiades images

Fusion of MS and hyperspectral images

(a) Reference (b) HS (c) MS

Figure : (a) Reference (b) Hyperspectral Image (size: $16 \times 16 \times 115$ Resolution: $20\text{m} \times 20\text{m}$) (c) Multispectral Image (size: $64 \times 64 \times 4$ Resolution: $5\text{m} \times 5\text{m}$)

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