Non-negative CP decomposition of hyperspectral data

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Outline

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Goals

1. Analysis of hyperspectral data by means of Canonical Polyadic (CP) tensor decomposition.
    1.1 Time-series.
    1.2 Multi-angle acquisitions.
    1.3 Conventional images.
2. Physical interpretation of the one-rank factors in terms of spectral unmixing.
3. Applications :
    3.1 Snow cover maps of the Alps : collaboration with LTHE laboratory and MeteoFrance.
    3.2 Analysis of Martian surface : collaboration with IPAG (Mars-ReCo project).

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Hyperspectral tensors

> Hyperspectral matrix : $\mathbf{X}^{I \times J}$.
  - I : number of pixels (spatial way).
  - J : number of bands (spectral way).

> Hyperspectral tensor : $\boldsymbol{\mathcal{X}}^{I \times J \times K}$.
  - K : number of time acquisitions (temporal way : SNOW project).
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Non-negative CP decomposition

> Formulation :

$$\mathcal{X}_{ijk} \approx \sum_{r=1}^{R} A_{ir} B_{jr} C_{kr} \lambda_r. \quad (1)$$

- $R \in \mathbb{N}_+$: tensor non-negative rank.
- $\mathbf{A}^{I \times R}$: spatial factors.
- $\mathbf{B}^{J \times R}$: spectral factors.
- $\mathbf{C}^{K \times R}$: temporal/angular factors.
- $\mathbf{\Lambda}^{R \times R}$: scaling diagonal matrix.
- **Everything is non-negative !**

> Compact representation : $\boldsymbol{\mathcal{X}} \approx (\mathbf{A}, \mathbf{B}, \mathbf{C})\, \mathbf{\Lambda}$.

> Optimization problem :

$$\begin{array}{ll} \text{minimize} & \|\boldsymbol{\mathcal{X}} - (\mathbf{A}, \mathbf{B}, \mathbf{C})\, \mathbf{\Lambda}\|_F^2 \\ \text{w.r.t.} & \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{\Lambda} \\ \text{subject to} & \mathbf{A} \succeq 0, \mathbf{B} \succeq 0, \mathbf{C} \succeq 0 \end{array} \quad (2)$$

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Properties

> Best lower non-negative rank approximates always exist $\rightarrow$ The problem is well-posed.

> There exist upper bounds to the tensor rank, R_o, that ensure uniqueness -> But only for exact decompositions !

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Algorithms

- > The CP optimization problem is highly non-convex.
- > Yet many algorithms provide rather precise computation.
- > These algorithms can be divided into two main classes :
  - *All-at-once gradient-based descent, e.g.* : all CP parameters are updated at the same time using a gradient scheme and non-negativity constraints are implemented through barriers or soft penalizations.
  - *Alternating minimization* : the cost function is minimized in an alternating way for each factor ($\mathbf{A}$, $\mathbf{B}$ or $\mathbf{C}$) while the others are fixed.
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Compression-based NN-CP decomposition

General approach [1] :

1. Compress the original tensor $\boldsymbol{\mathcal{X}}^{I,J,K}$ on a small tensor $\boldsymbol{\mathcal{G}}^{R_1,R_2,R_3}$.
   - Supposition : the compressed tensor $\boldsymbol{\mathcal{G}}$ contains almost the same information as the original tensor $\boldsymbol{\mathcal{X}}$.
2. Compute the CP decomposition of $\boldsymbol{\mathcal{G}}$.
   - Consider non-negativity constraints of $\boldsymbol{\mathcal{X}}$ in the optimization problem.
   - Estimate the compressed factors $\mathbf{A}_c^{R_1 \times R}$, $\mathbf{B}_c^{R_2 \times R}$ and $\mathbf{C}_c^{R_3 \times R}$.
3. Uncompress the estimated one-rank factors $\mathbf{A}_c$, $\mathbf{B}_c$ and $\mathbf{C}_c$ to obtain the original factors $\mathbf{A}^{I \times R}$, $\mathbf{B}^{J \times R}$ and $\mathbf{C}^{K \times R}$.

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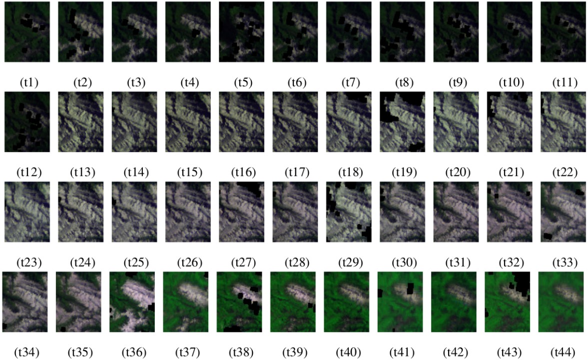
1 Motivation

2 Non-negative CP decomposition

3 Experimental results with time series

4 What about hyperspectral data cubes ?

- > 44 daily MODIS acquisitions with low cloud coverage.
- > Pre-processed to increase spatial resolution (250m).
- > Dimensions : $4800 \times 7 \times 44$.

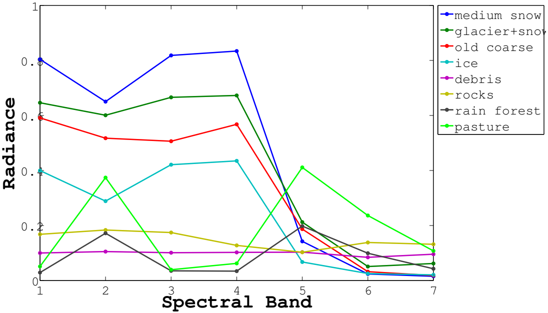


[2] Veganzones, M.A. ; Cohen, J. ; Cabral-Farias, R. ; Chanussot, J. ; Comon, P. ; "Nonnegative tensor CP decomposition of hyperspectral data," IEEE Transactions on Geoscience and Remote Sensing, 54(5), pp. 2577-2588, 2016.

Experimental methodology (I)

> Competing algorithms :
  - Compression-based CP algorithms : CCG and ProCoALS.
  - Conventional CP algorithm : ANLS.
  - Conventional day-basis full-constrained spectral unmixing (FCLSU).

> Groundtruth : 8 spectra (on-field acquisitions).



Experimental methodology (II)

- > We run 50 Monte Carlo runs for each of the algorithms for a set of different rank values in the range $R \in [5, 15]$.
- > For the compression-based CP algorithms, the compressed tensor, $\boldsymbol{\mathcal{X}}_c$, has dimensions $175 \times 7 \times 25$.
- > Quality measures :
  - Reconstruction error : average RMSE.
  - Angular error w.r.t. the groundtruth spectral signatures.
  - Linear Pearson correlation w.r.t. the spatial abundances obtained by the FCLSU.

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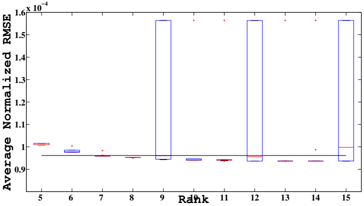
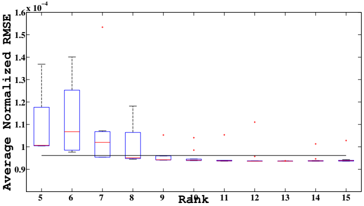
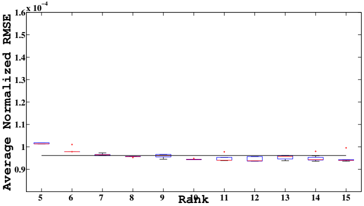
Reconstruction errors



(a) ANLS

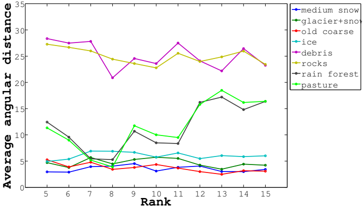


(b) CCG



(c) ProCoALS

Spectral factors interpretation



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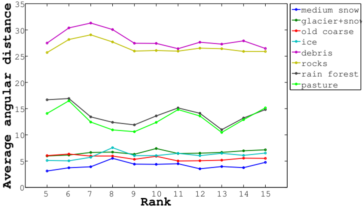

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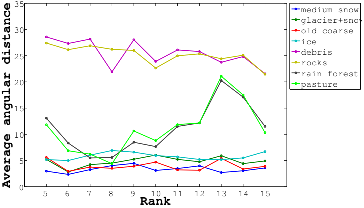

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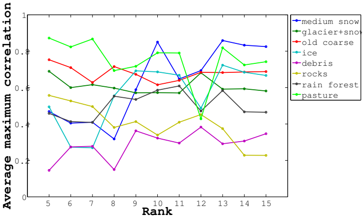
Spatial factors interpretation



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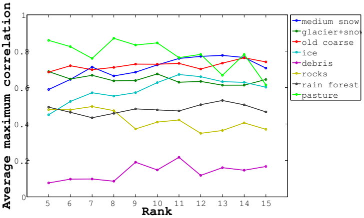

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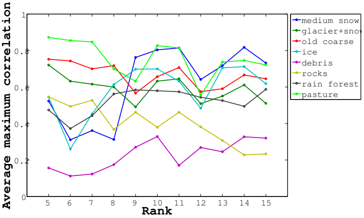

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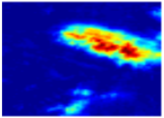
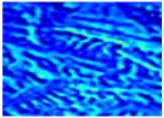
Spatial factors interpretation

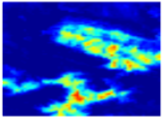
> Best run with rank $R = 8$.



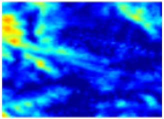
Permanent snow/ice



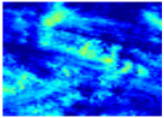
Seasonal snow/ice (type 1)



Seasonal snow/ice (type 2)



Seasonal vegetation (type 1)



Seasonal vegetation (type 2)

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Are hyperspectral data cubes tensors ?

- Hyperspectral images are often provided, not as non-negative matrices, $\mathbf{X} \in \mathbb{R}_{+}^{I \times J}$, but as data cubes, $\boldsymbol{\mathcal{X}} \in \mathbb{R}_{+}^{I_r \times I_c \times J}$, where I_r and I_c denote the number of rows and columns respectively, being $I = I_r I_c$.
- It is possible to think of the data cube as a tensor representation of the image, allowing to apply tensor analysis techniques.
- This is fine ! $\rightarrow$ Employed for de-noising, compression, ...
- ... but, in general, it is not possible to suppose a low rank, that is, a small R.
- Proposed solution : patch-tensors [3].

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