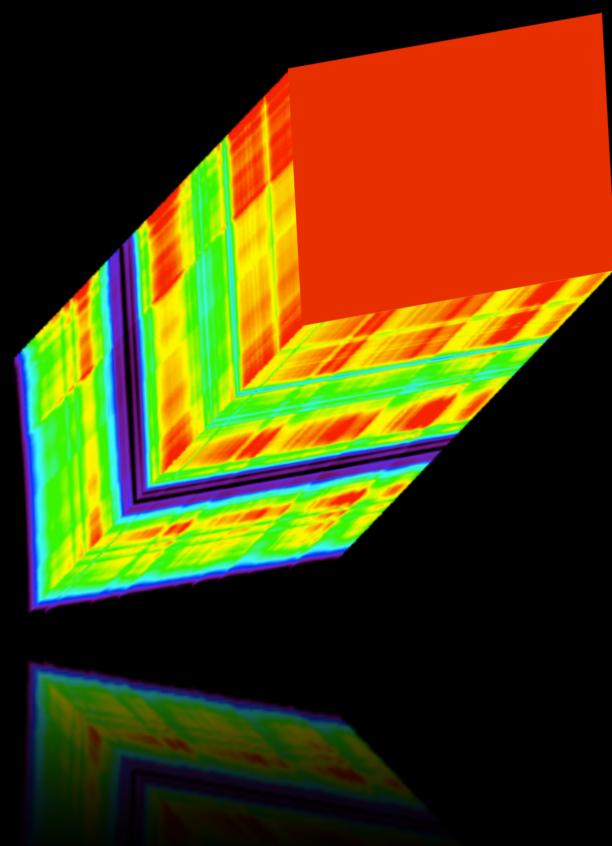
Inversion bayésienne avec tables de données



F. Andrieu [1], F. Schmidt [1]

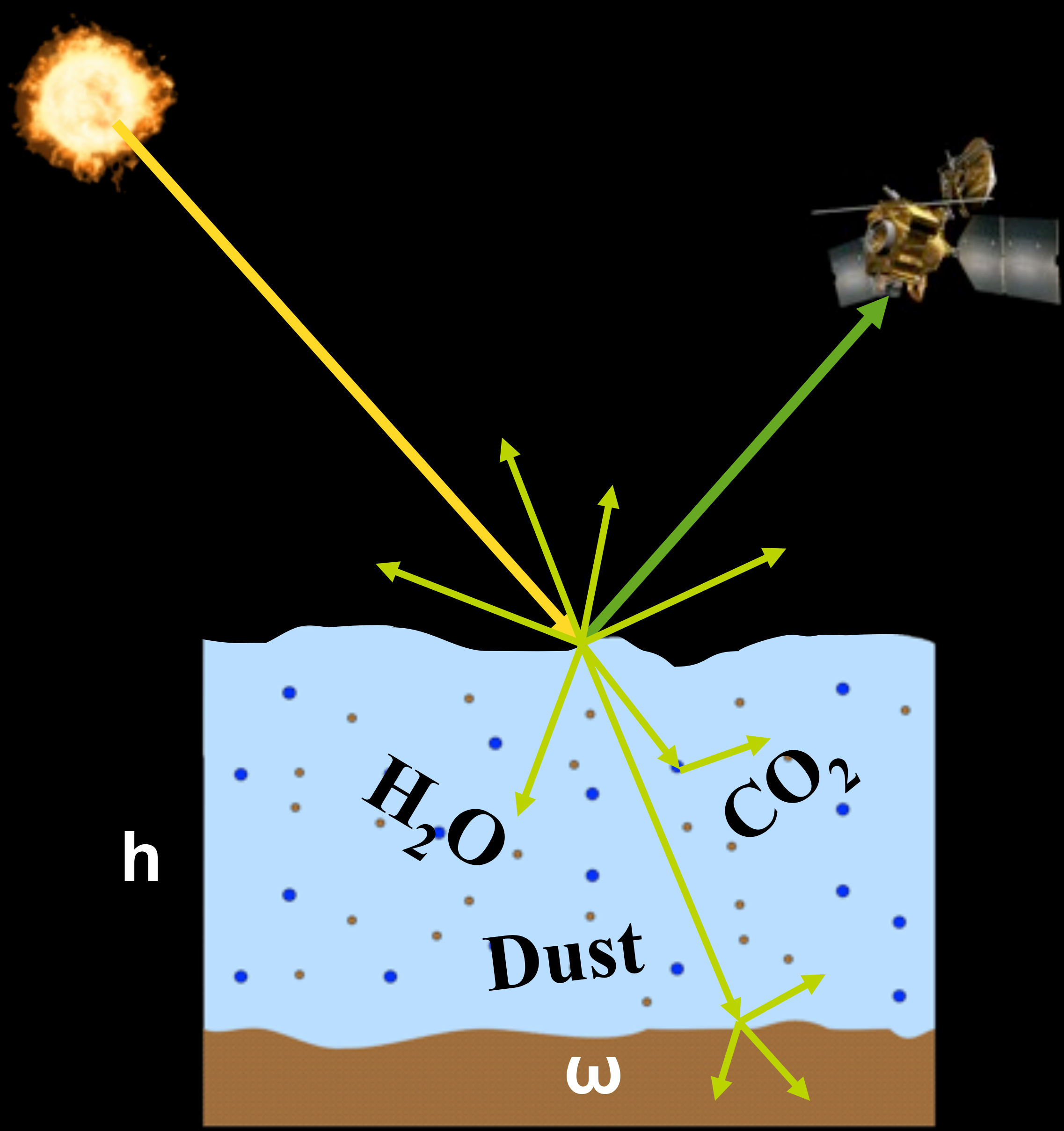
[1] GEOPS, Univ. Paris-Sud, CNRS, Université Paris-Saclay, Rue du Belvédère, Bât. 509, 91405 Orsay, France

Colloque de la sfpt-gh, Grenoble, 11-13 mai 2016

Model

[Set of parameters] ➡ spectrum

m $F(m)$

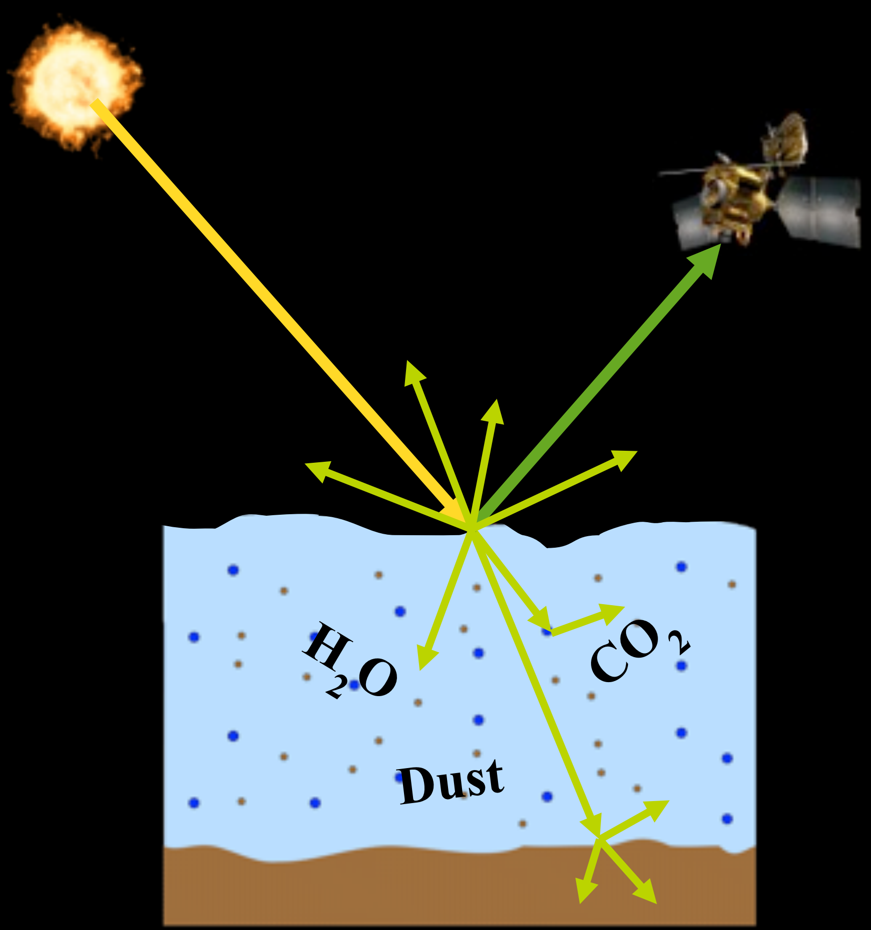


$$F(m) = R_\lambda(h,\ \omega,\ \varnothing_{H20}, p_{H2O}, \varnothing_{CO2}, p_{CO2})$$

Complexity

Instability

Inversion



Direct model

[Set of parameters] ➡ spectrum

m $F(m)$

Inversion

spectrum

d_{mes} → [Set of parameters] m

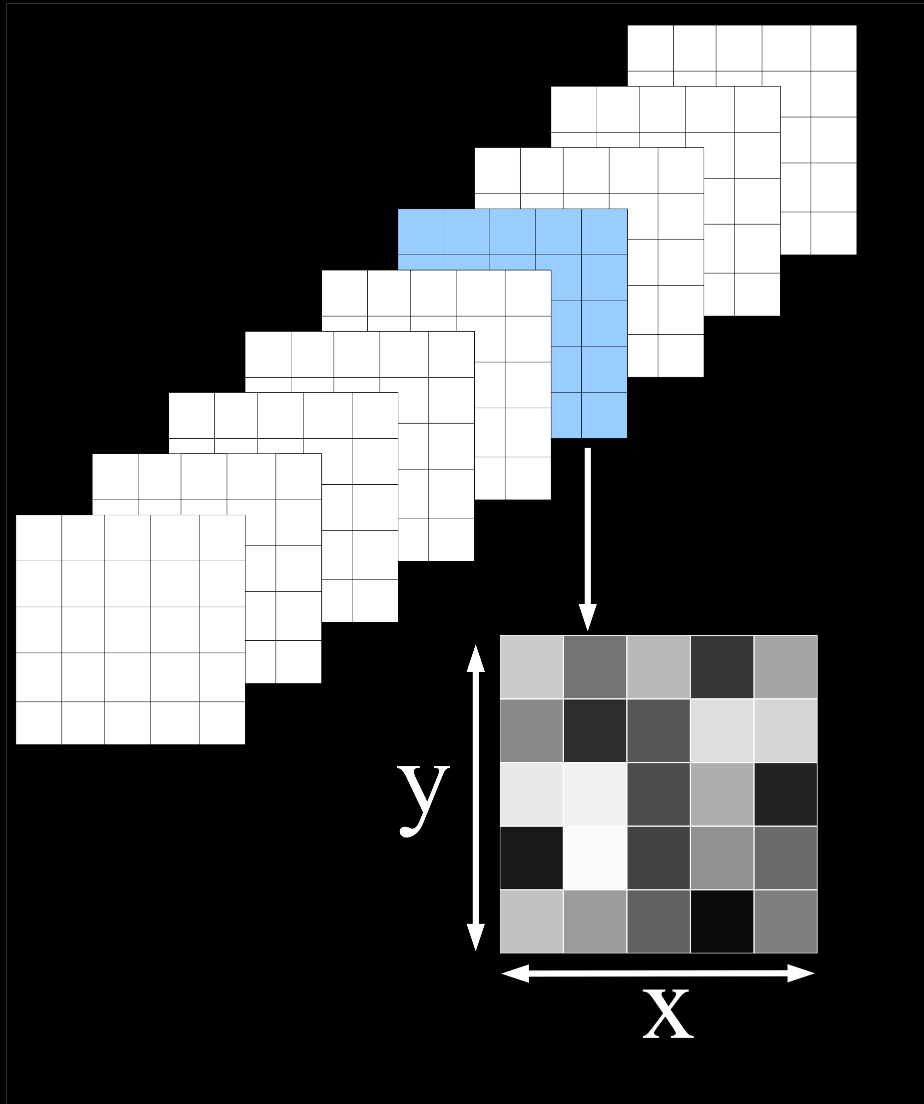
model

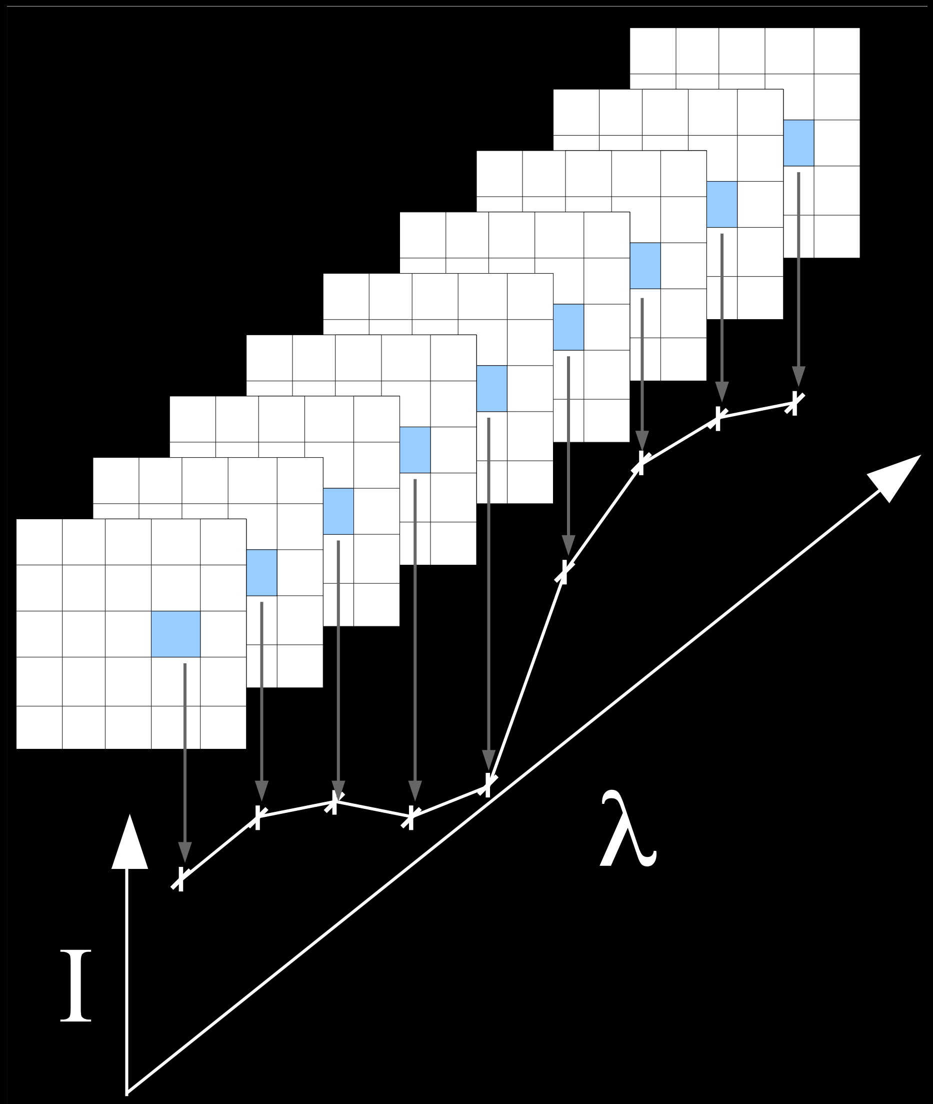
$F(m)$ → m

Satellite hyperspectral data = massive datasets

CRISM > 22 TB (10^{10} spectres)

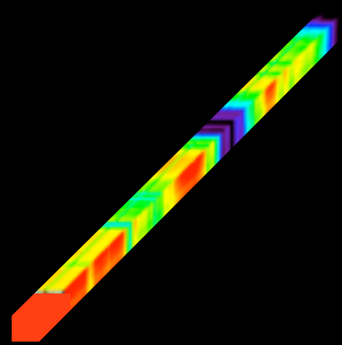
source JHU/APL 10/2015





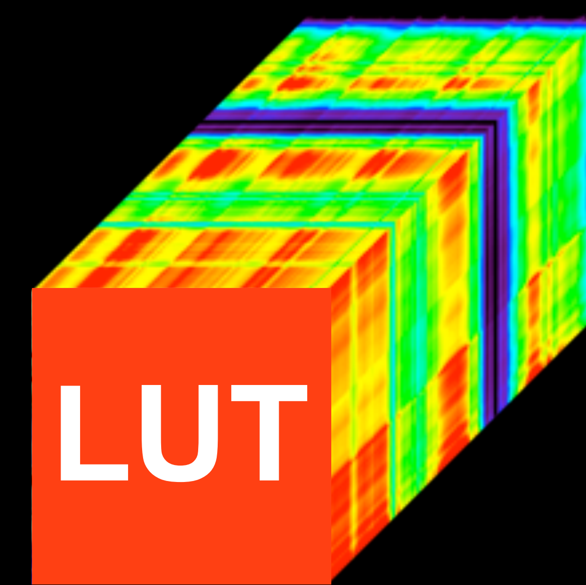
Inversion method: FAST

Fast → Comparison with LUT



Spectroscopic or spectro-imaging **data**

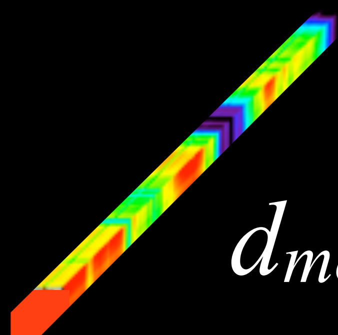
↔



Spectral database (**model** LUT)

Nearest neighbor(s) methods :

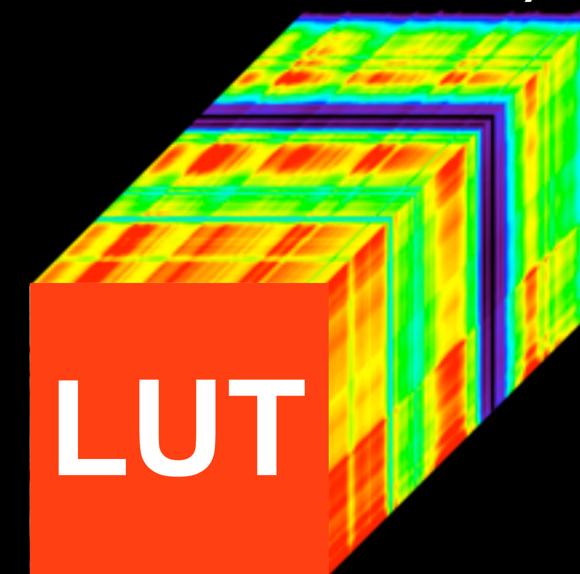
Spectroscopic or spectro-imaging **data**



d_{mes}

Spectral database (**model** LUT)

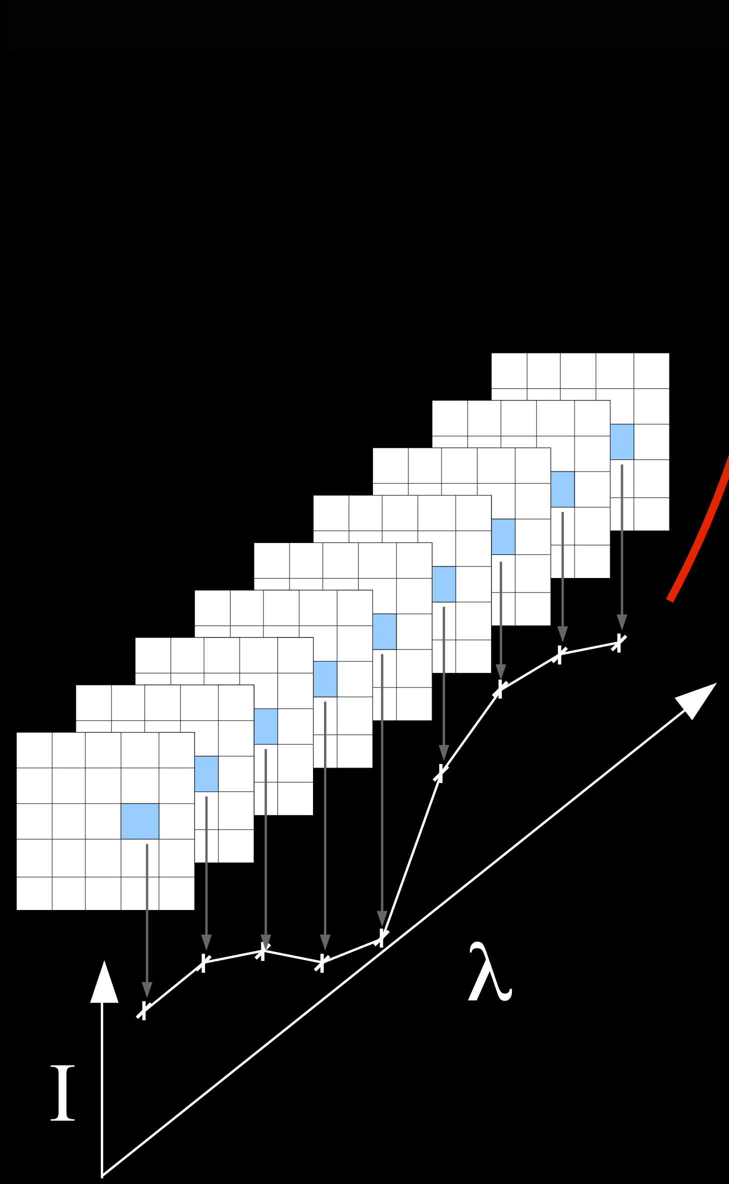
$\{F(m_i)\}$



$$D\left(d_{mes}, \mathrm{F}\left(m_i\right)\right) = \left\|d_{mes} - \mathrm{F}\left(m_i\right)\right\|_2 = \sqrt{\left(d_{mes} - \mathrm{F}\left(m_i\right)\right)^T \cdot \left(d_{mes} - \mathrm{F}\left(m_i\right)\right)}$$

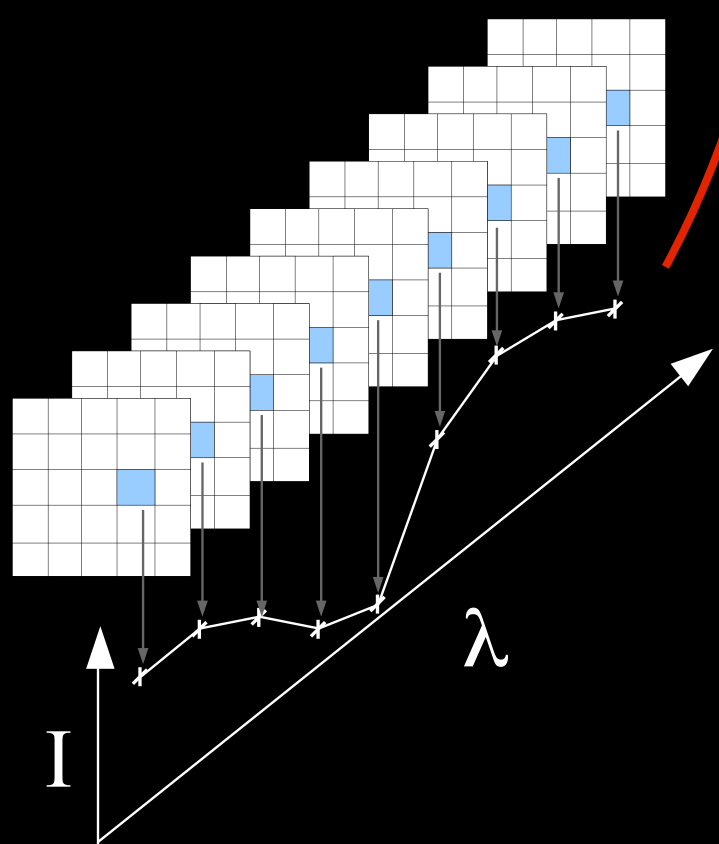
Nearest neighbor(s) methods :

$$D\left(d_{mes},\mathrm{F}\left(m_i\right)\right) = \left\| d_{mes} - \mathrm{F}\left(m_i\right)\right\|_2 = \sqrt{\left(d_{mes} - \mathrm{F}\left(m_i\right)\right)^T \cdot \left(d_{mes} - \mathrm{F}\left(m_i\right)\right)}$$

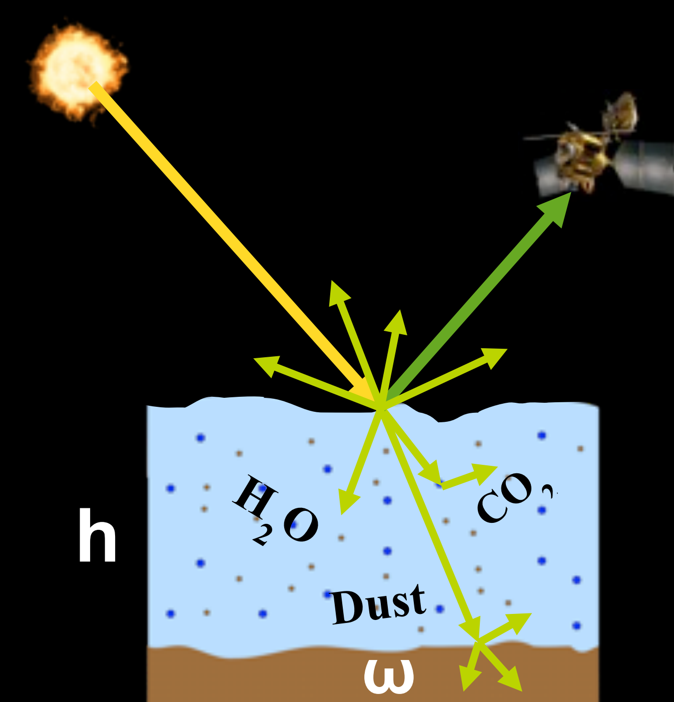


Nearest neighbor(s) methods :

$$D(d_{mes},\mathrm{F}(m_i)) = \|d_{mes} - \mathrm{F}(m_i)\|_2 = \sqrt{(d_{mes}-\mathrm{F}(m_i))^T \cdot (d_{mes}-\mathrm{F}(m_i))}$$



$\mathbf{R(h, \omega, \varnothing_{H20}, p_{H2O}, \varnothing_{CO2}, p_{CO2})}$



Nearest neighbor(s) methods : variants

- $D(d_{mes}, \mathrm{F}(m_i)) = \|d_{mes} - \mathrm{F}(m_i)\|_2 = \sqrt{(d_{mes} - \mathrm{F}(m_i))^T \cdot (d_{mes} - \mathrm{F}(m_i))}$

- $D(d_{mes}, \mathrm{F}(m_i)) = \sqrt{(d_{mes} - \mathrm{F}(m_i))^T \overline{\overline{M}} (d_{mes} - \mathrm{F}(m_i))}$

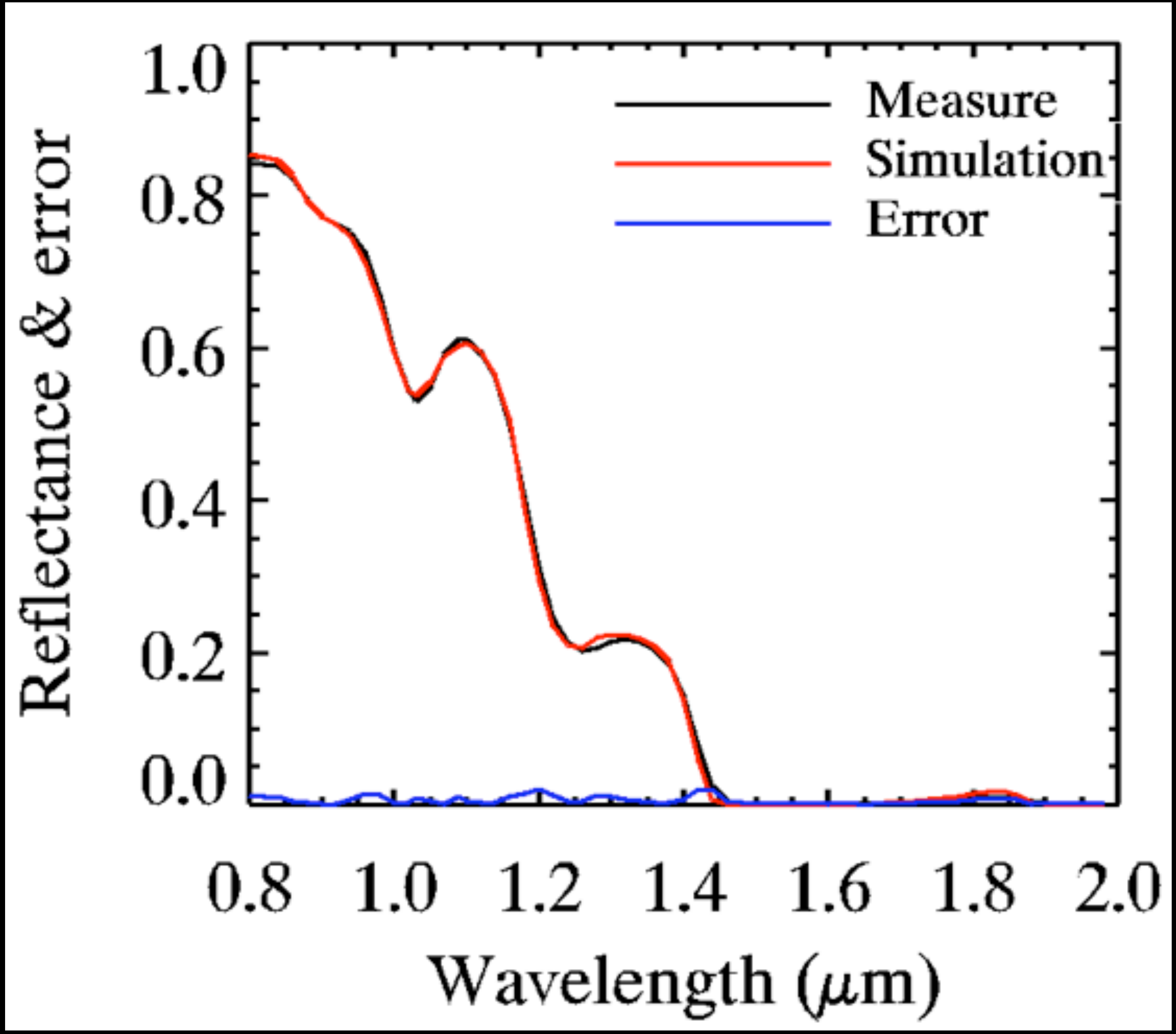
- $corr(d_{mes}, \mathrm{F}(m_i)) = \dfrac{(d_{mes})^T \cdot \mathrm{F}(m_i)}{\|d_{mes}\|_2 \times \|\mathrm{F}(m_i)\|_2}$

d_{mes}	measure
m_i	set of model parameters
$\mathrm{F}(m_i)$	simulation

- SFF (*Spectral Feature Fitting*) *Clark et al., 1990, van der Meer , 2004*

Nearest neighbor(s) methods: result

- m_r : Best fit (1 or several)
- residual (RMS...)



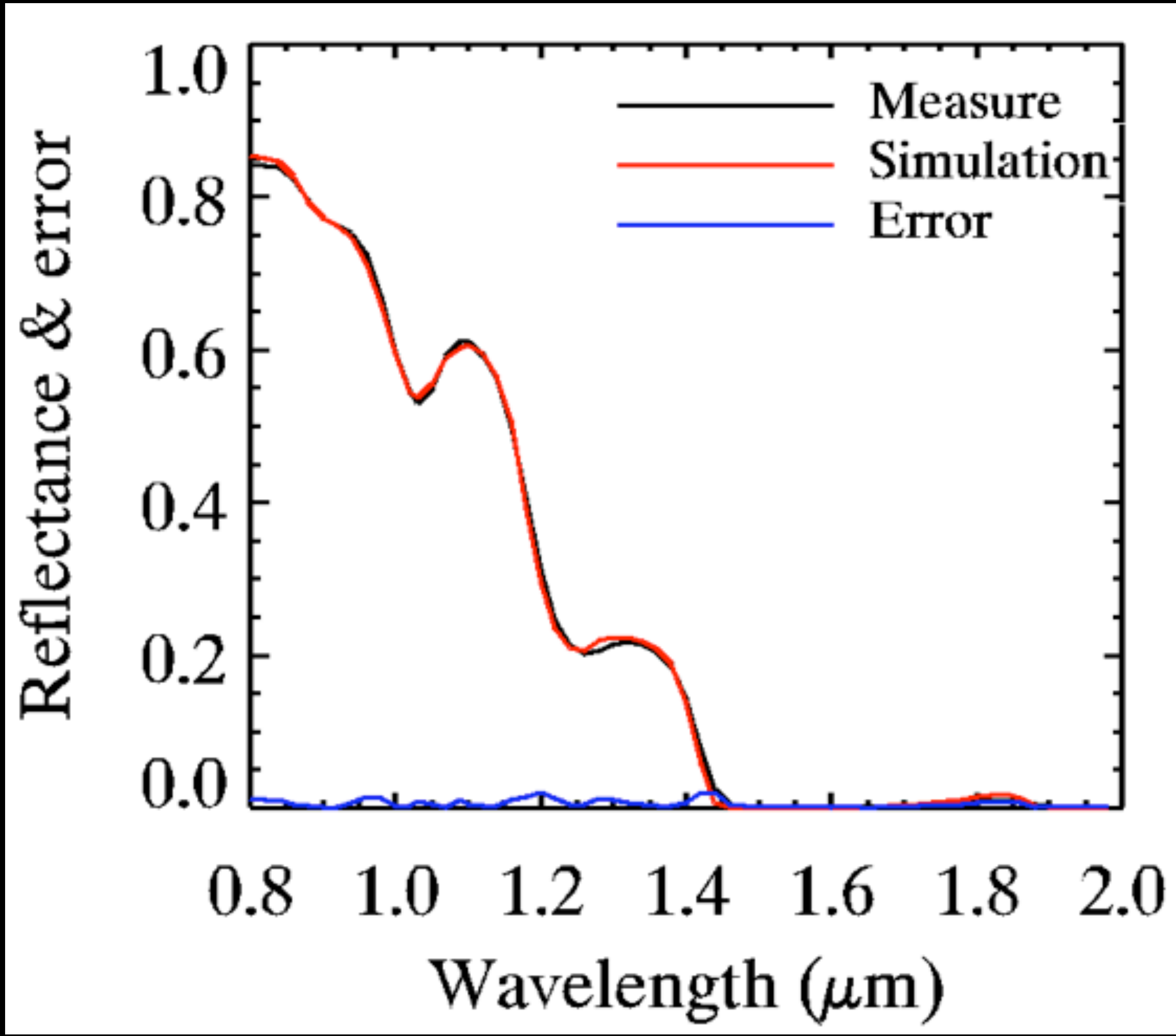
➡ Definition of «good fit»?

➡ How many «good fits»?

➡ What information on m_r?

Nearest neighbor(s) methods: result

- m_r : Best fit (1 or several)
- residual (RMS...)



➡ Definition of «good fit»?

➡ How many «good fits»?

➡ What information on m_r?

NOT ENOUGH !

Solution: Bayesian method

Bayesian : control of uncertainties

~~Quantity = Value~~

Quantity = Probability Density Function (PDF)

d_{mes}	measure
m_i	set of model parameters
$\mathrm{F}(m_i)$	simulation

d_{mes} : random realisation of *n* dimensionnal *PDF* $\rho_D(d)$

d_{mes} → $\pi_D(d_{mes} \mid m)$ ← $\rho_D(d)$ parametric model

Tarantola & Valette, 1982

Bayesian method :

$$\mathscr{P}(A \mid B) = \frac{\mathscr{P}(A)\,\mathscr{P}(B \mid A)}{\mathscr{P}(B)}$$

$$\rho_M(m \mid d_{mes}) = \frac{\pi_D(d_{mes} \mid m)\,\rho_M(m)}{\int_M \pi_D(d_{mes} \mid x)\,\rho_M(x)\,\mathrm{d}x}$$

d_{mes}	measure
m_i	set of model parameters
$\mathrm{F}(m_i)$	simulation
$\rho_M(m)$	a priori PDF
$\rho_M(m \mid d_{mes})$ $\sigma_M(m)$	a posteriori PDF
$\pi_D(d_{mes} \mid m)$	parametric model
$\overline{\overline{C}}$	covariance matrix

Under gaussian hypothesis :

$$\sigma_M(m) = \mathrm{L}(m)\,\rho_M(m)\,k$$

$$\mathrm{L}(m) = \exp\left(-\frac{1}{2} \times (\mathrm{F}(m) - d_{mes})^T\,\overline{\overline{C}}^{-1}\,(\mathrm{F}(m) - d_{mes})\right)$$

Tarantola & Valette, 1982

Solution: Bayesian method

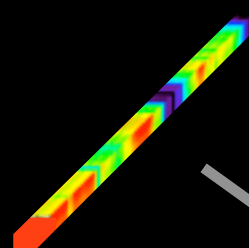
d_{mes} measure

m_i set of model parameters

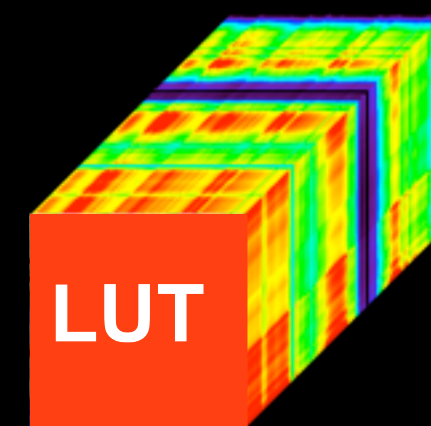
$\mathrm{F}(m_i)$ simulation

$\overline{\overline{C}}$ covariance matrix

Spectroscopic or spectro-imaging **data**



uncertainties on the data



Model LUT

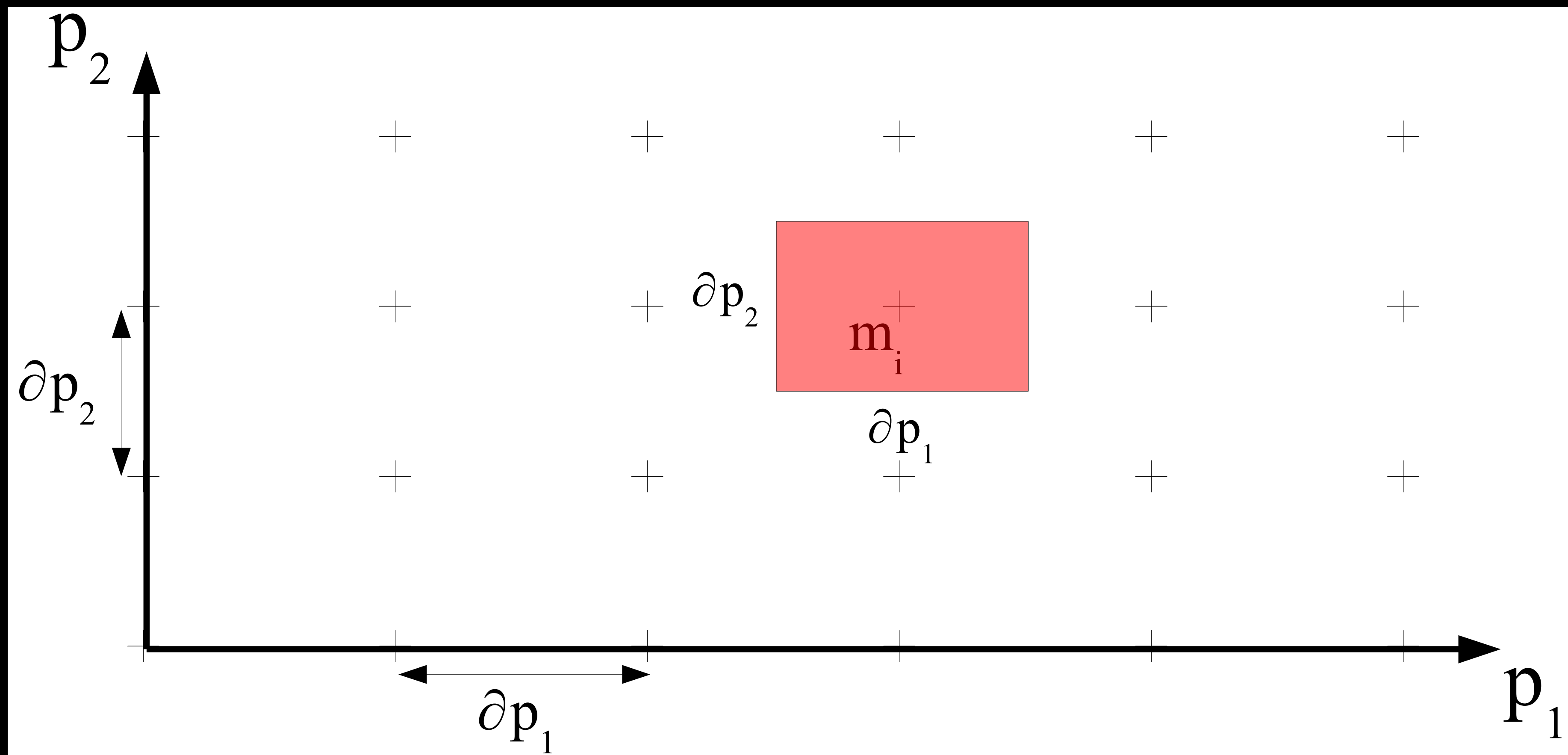
$$\mathrm{L}(i) = \exp\left(-\frac{1}{2} \times \left(\mathrm{F}(m_i) - d_{mes}\right)^T \overline{\overline{C}}^{-1} \left(\mathrm{F}(m_i) - d_{mes}\right)\right)$$

Likelihood

→ **Uncertainties on the result**

Tarantola & Valette, 1982

Bayesian inversion method



$$\mathscr{P}\{m_i\} = \frac{\mathrm{L}(m_i)\,\mathrm{d}p(m_i)}{\sum_i \mathrm{L}(m_i)\,\mathrm{d}p(m_i)}$$

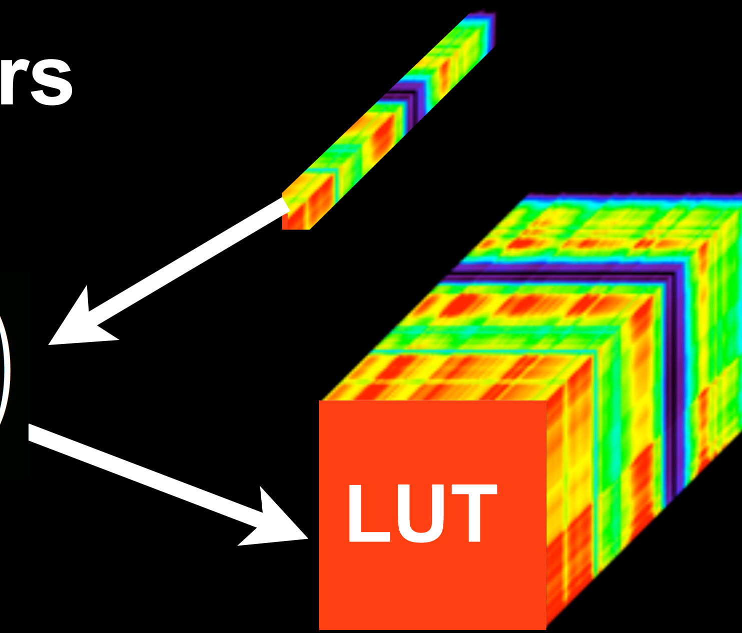
$$\mathscr{P}\{p_j(k)\} = \frac{\mathrm{L}'(k)}{\sum_i \mathrm{L}'(k)\,\partial p_j(k)}$$

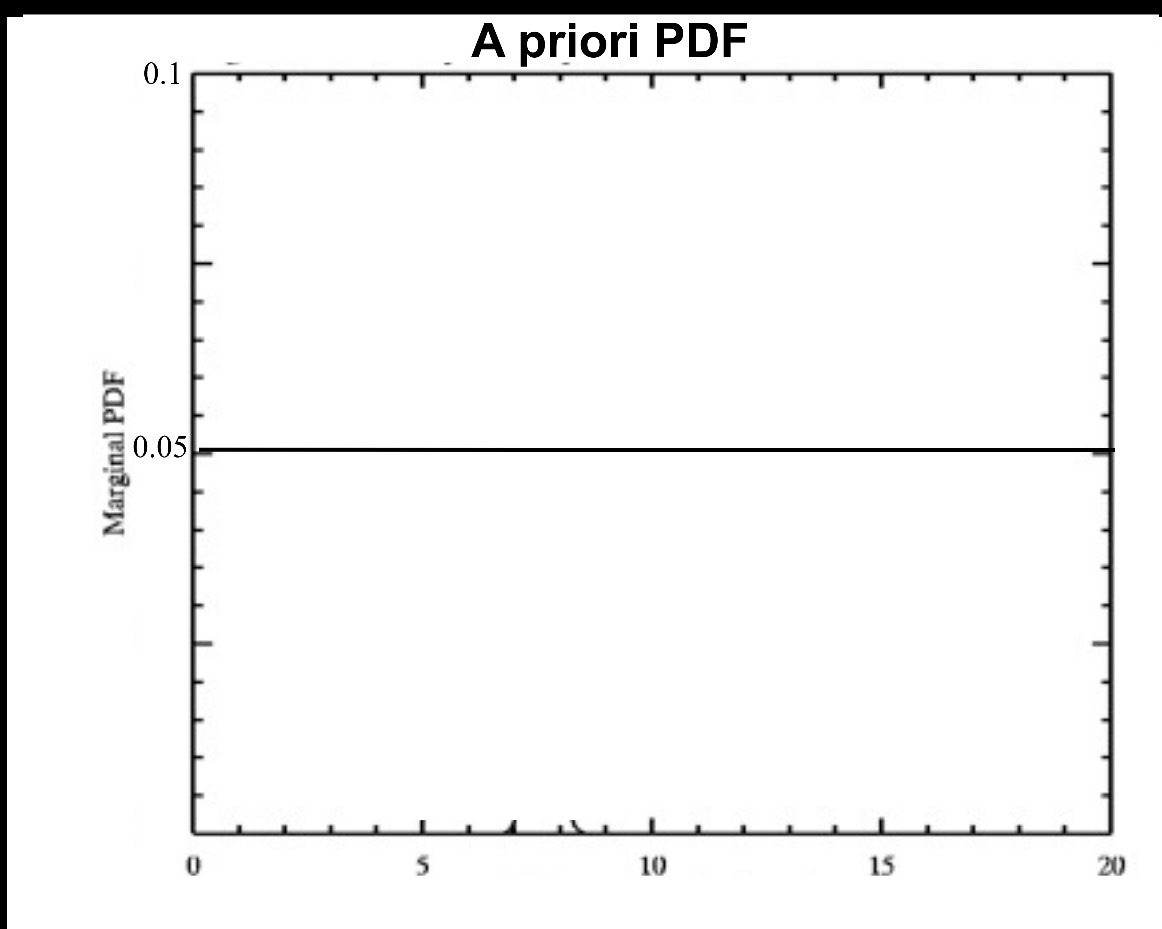
$$\mathrm{L}'(k) = \sum_i \mathrm{L}(m_i \mid p_j(k)) \prod_{l \neq j} \partial p_l(m_i \mid p_j(k))$$

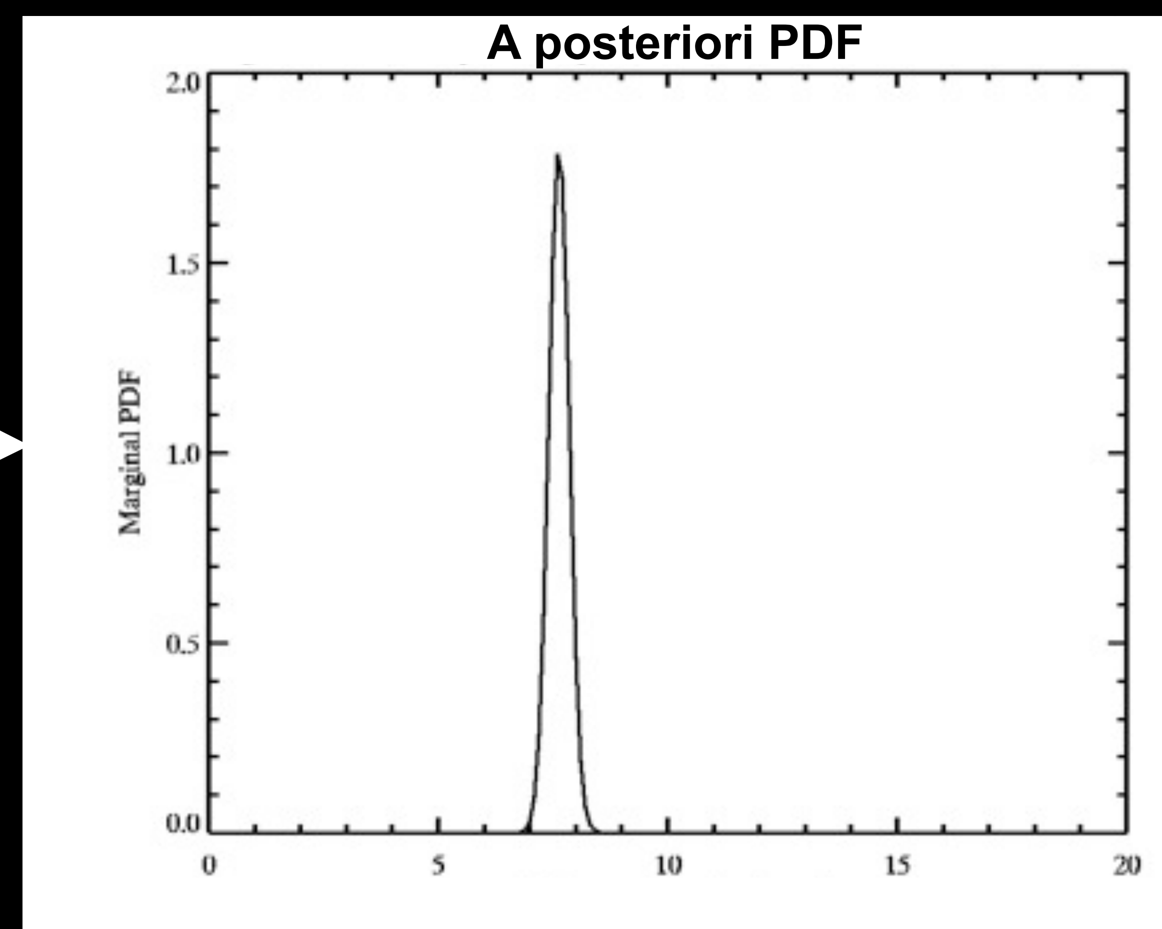
Bayesian inversion method:

Sampling a posteriori PDF for the parameters

$$L(i) = \exp\left(-\frac{1}{2} \times \left(F(m_i) - d_{mes}\right)^T \overline{\overline{C}}^{-1} \left(F(m_i) - d_{mes}\right)\right)$$







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Sampling a posteriori PDF for the parameters

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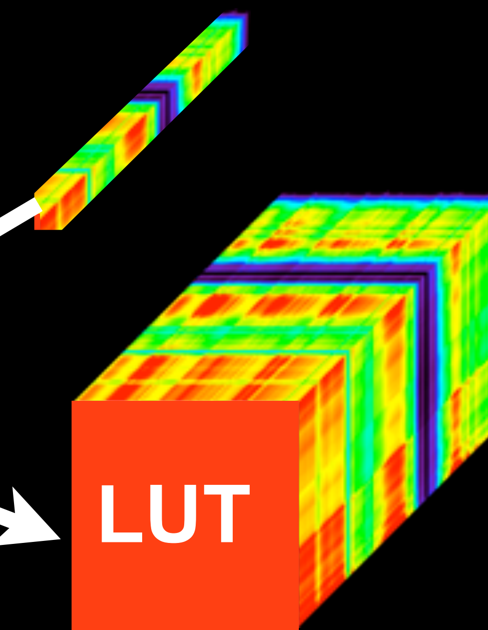

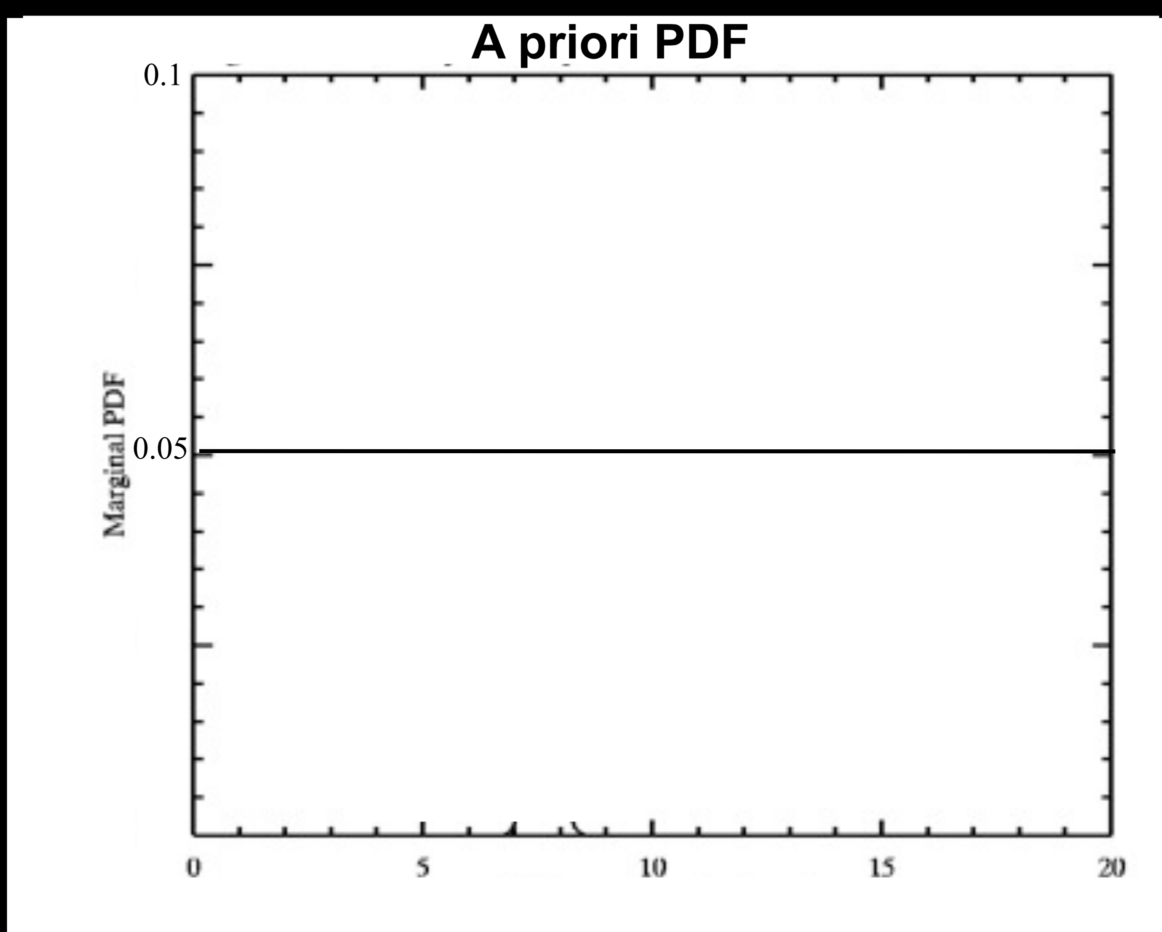

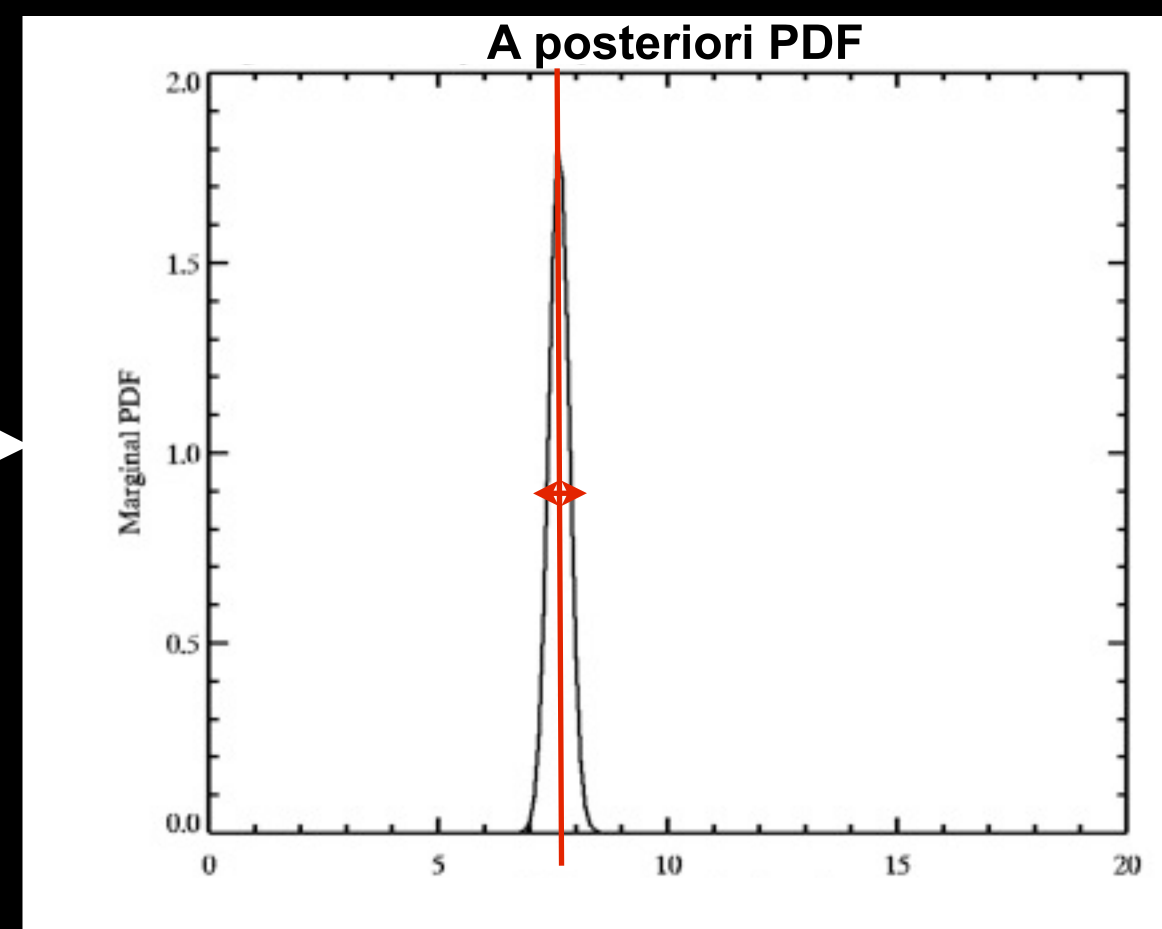

Bayesian inversion method:

Sampling a posteriori PDF for the parameters

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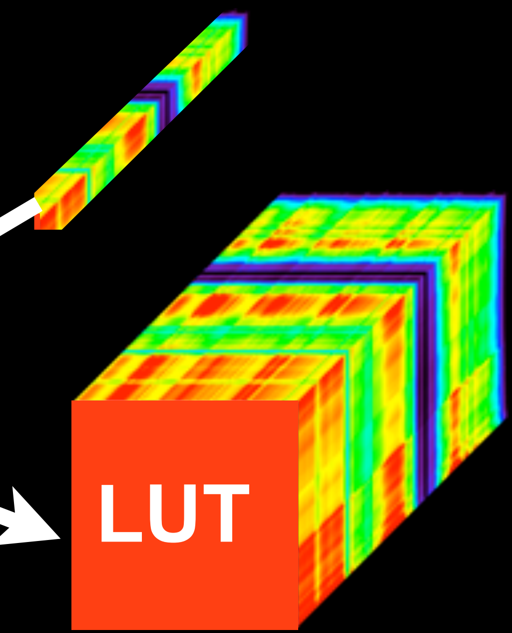

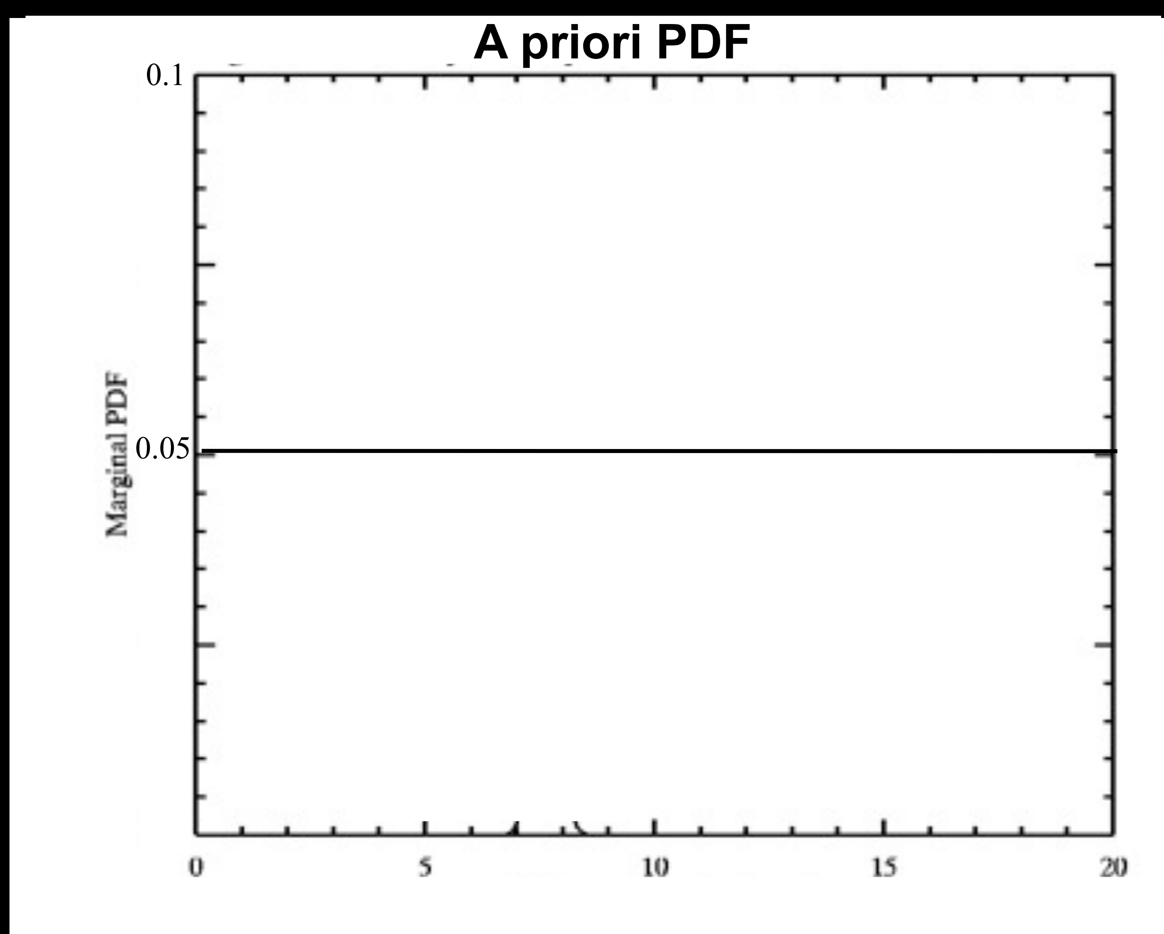

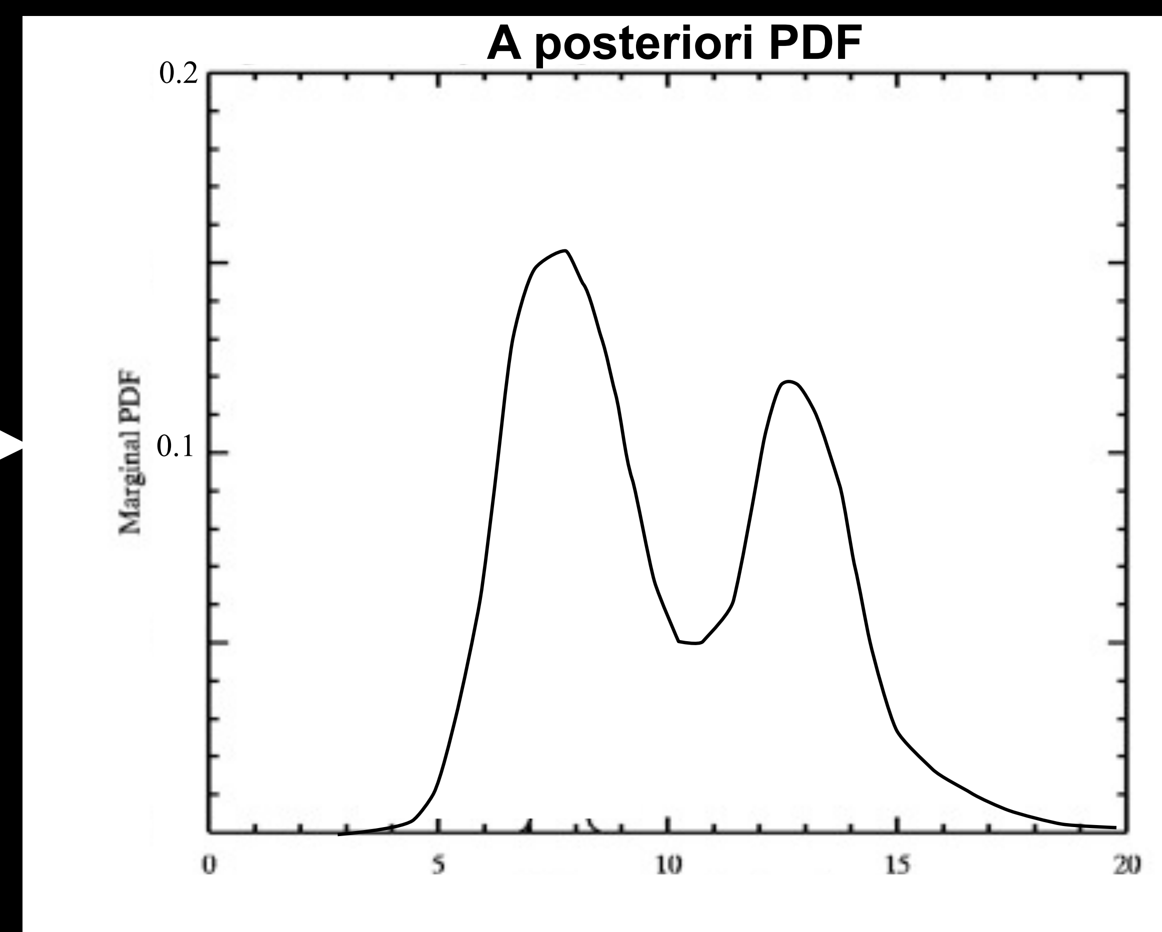

Bayesian inversion method:

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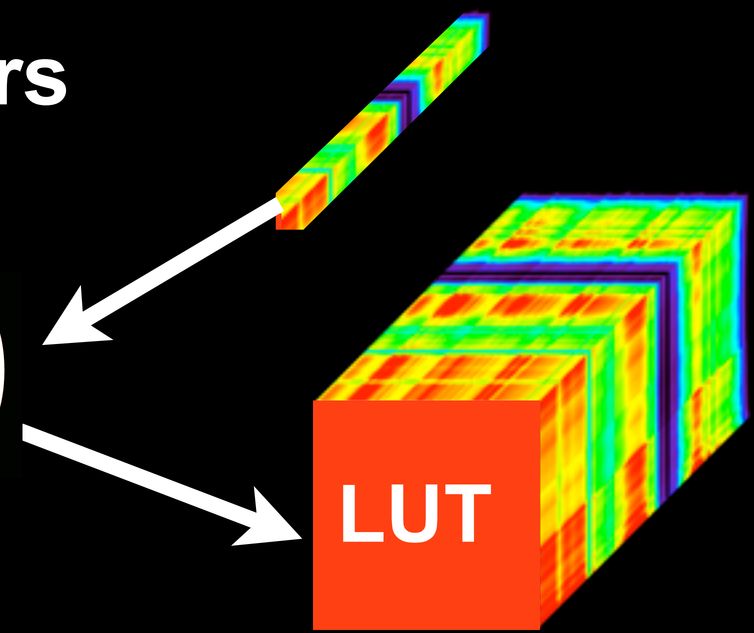
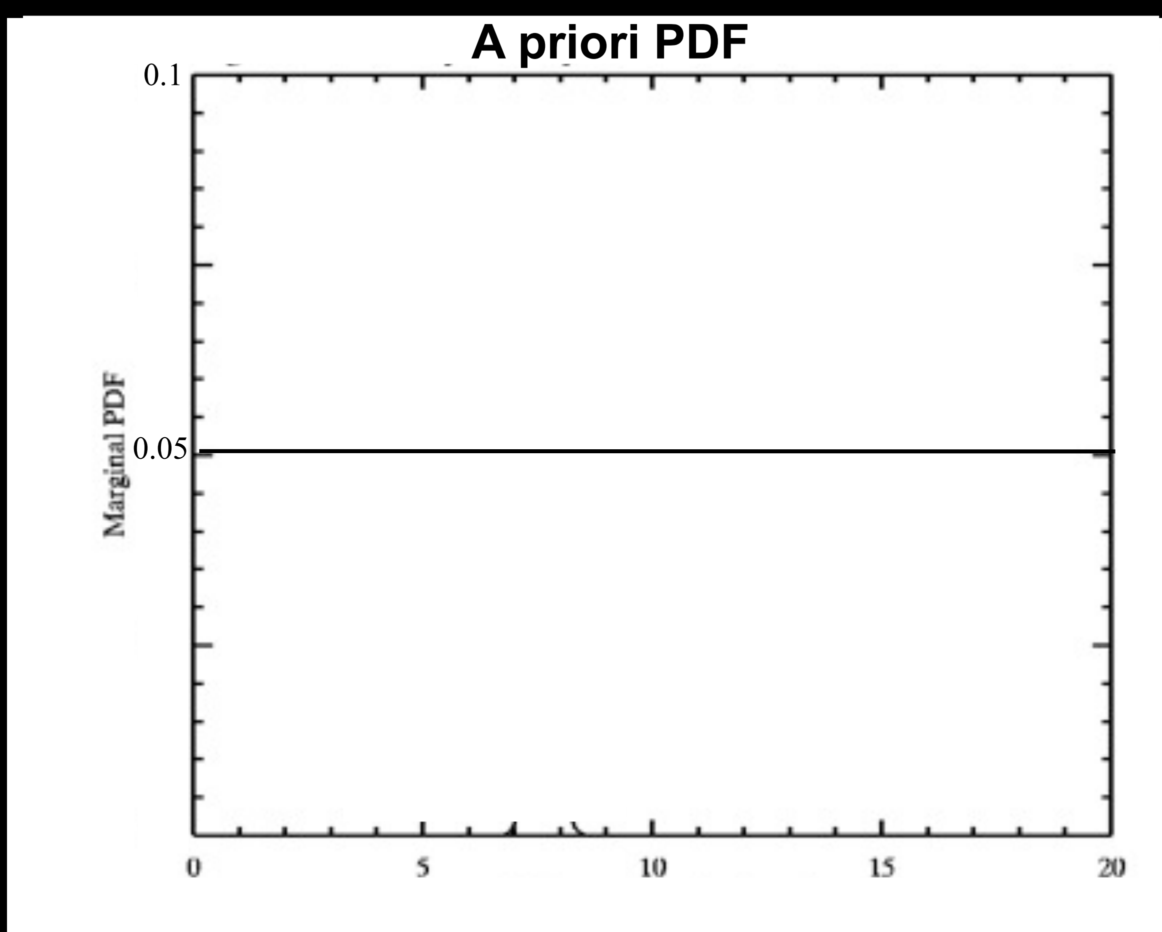
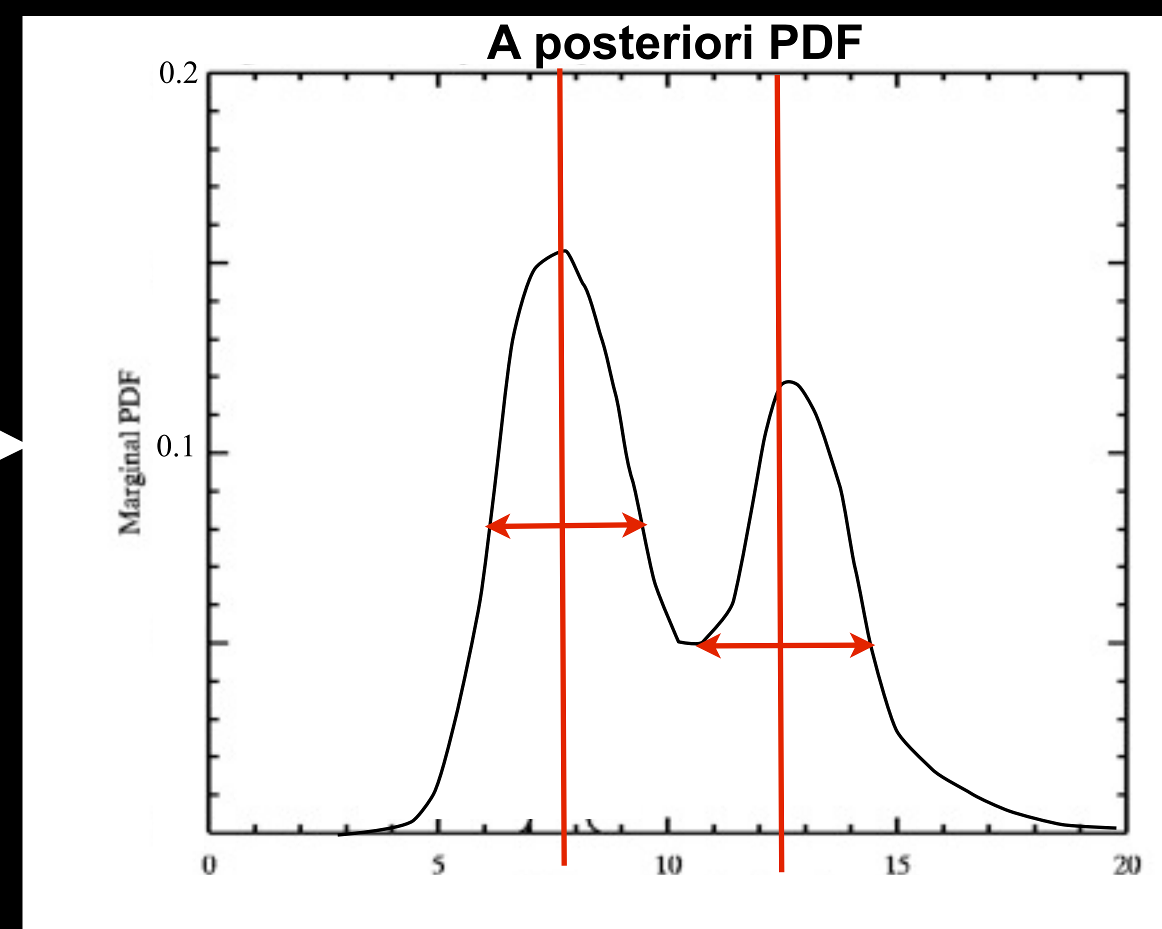




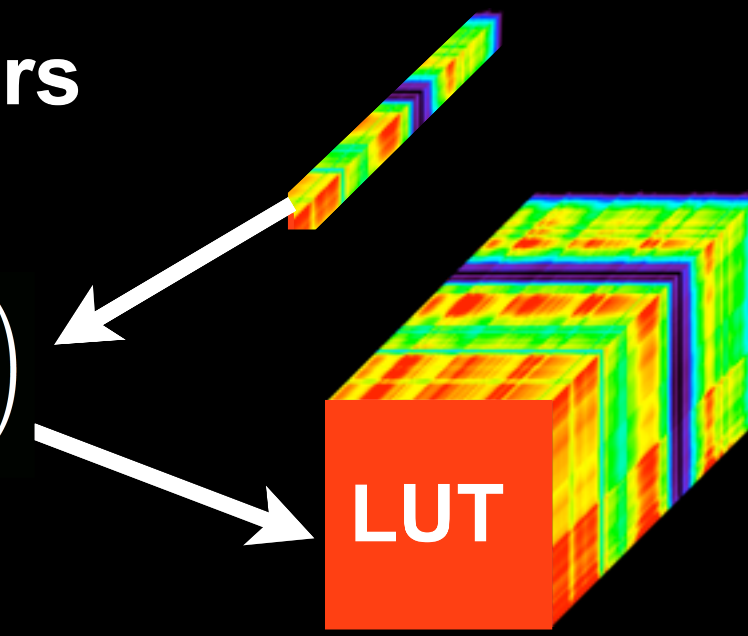


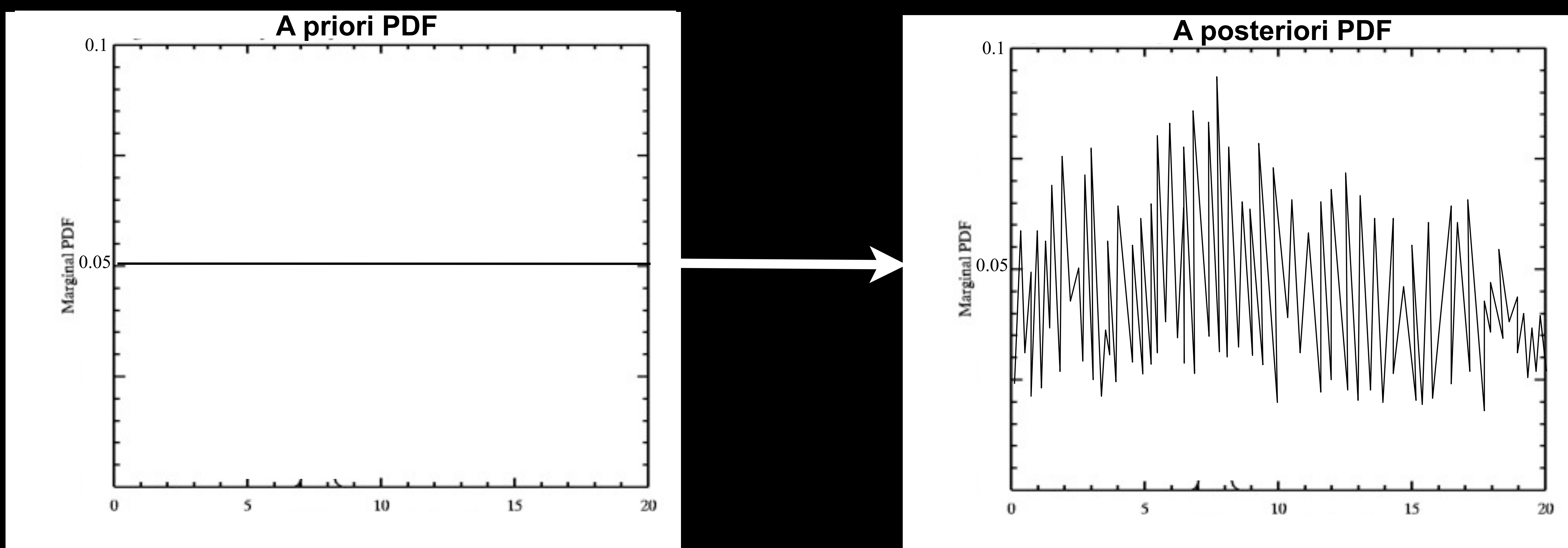
jeudi 12 mai 16

Bayesian inversion method:

Sampling a posteriori PDF for the parameters

$$\mathrm{L}(i) = \exp\left(-\frac{1}{2} \times (\mathrm{F}(m_i) - d_{mes})^T \,\overline{\overline{C}}^{-1} (\mathrm{F}(m_i) - d_{mes})\right)$$





Bayesian inversion method:

Sampling a posteriori PDF for the parameters

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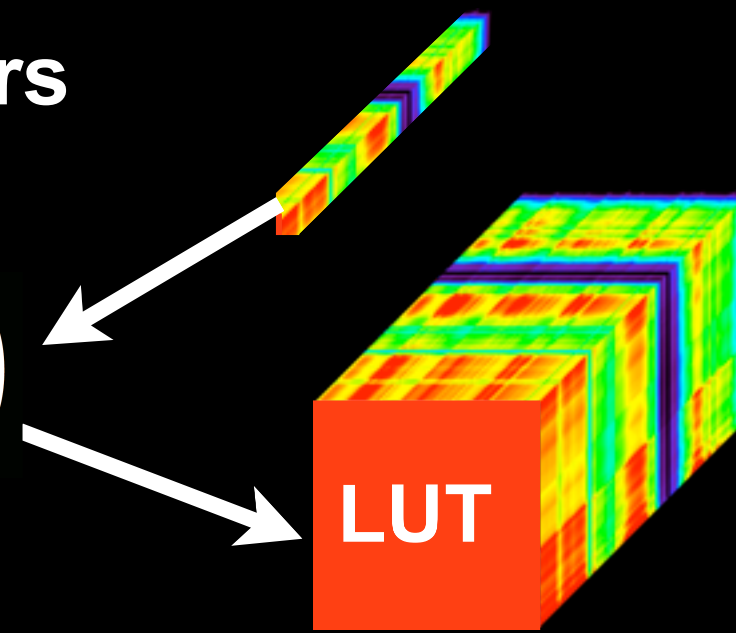
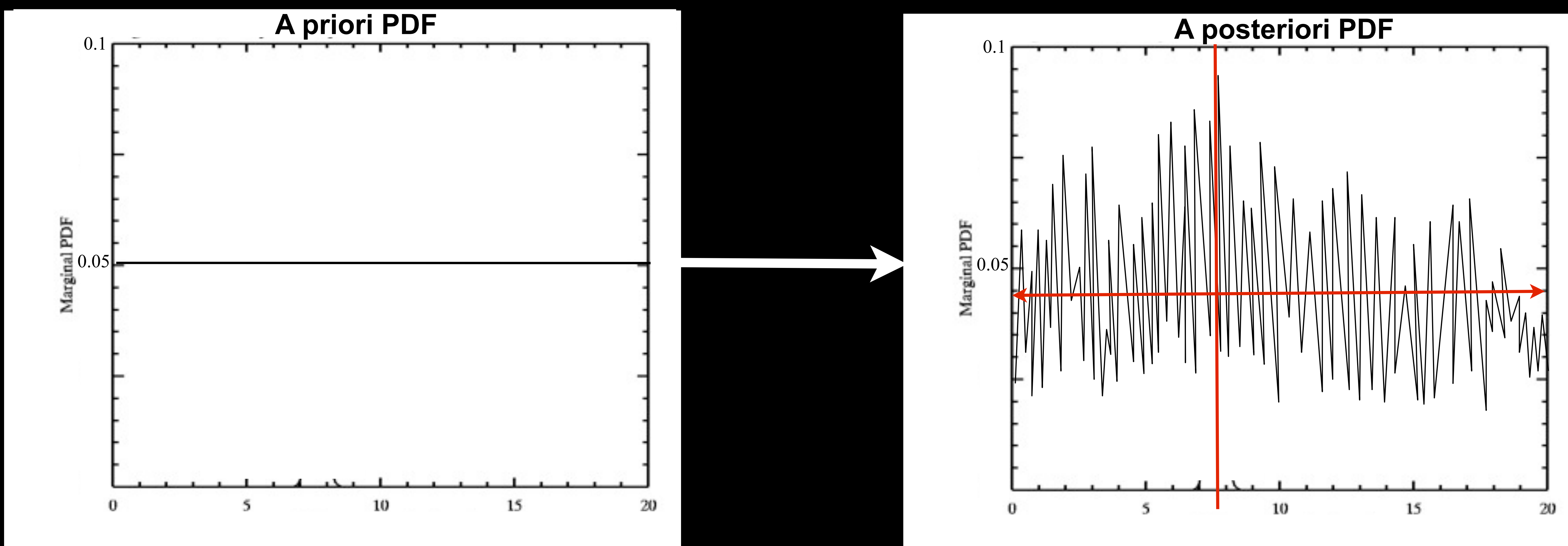
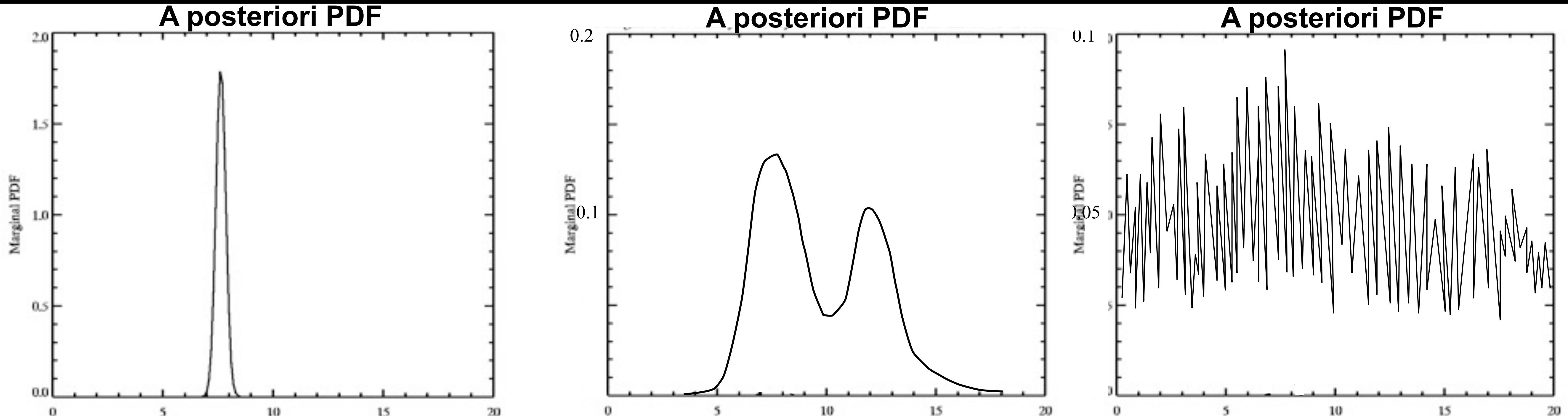




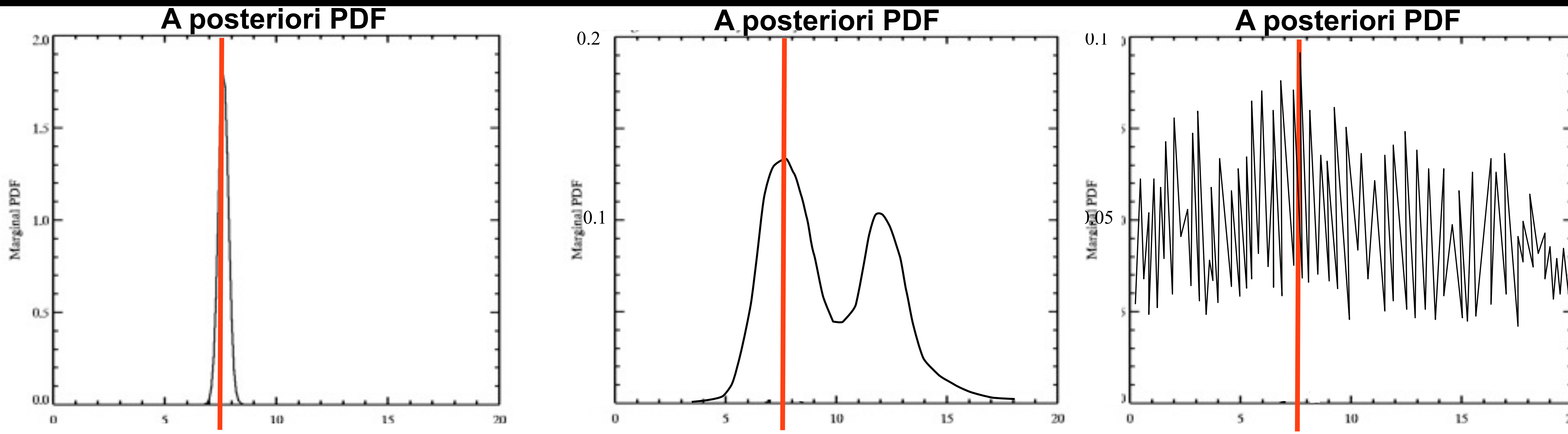
Bayesian inversion method:

Same best fit



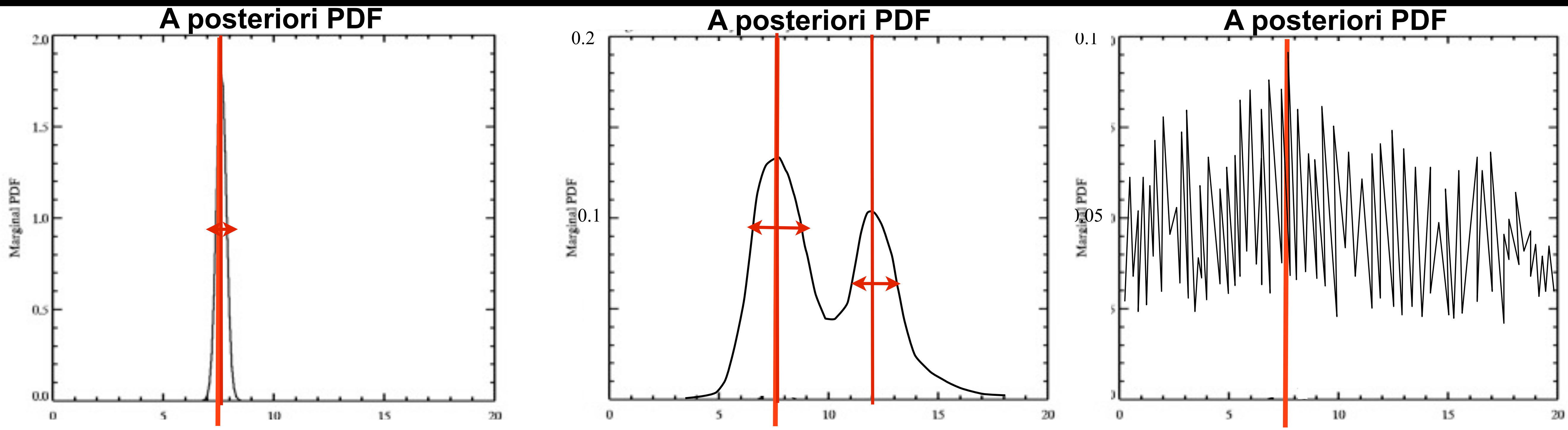
Bayesian inversion method:

Same best fit



Bayesian inversion method:

Same best fit



Reliable result | **Ambiguous result** | **No information**

$\langle r \rangle \pm 2.\sigma$ | **further investigation**

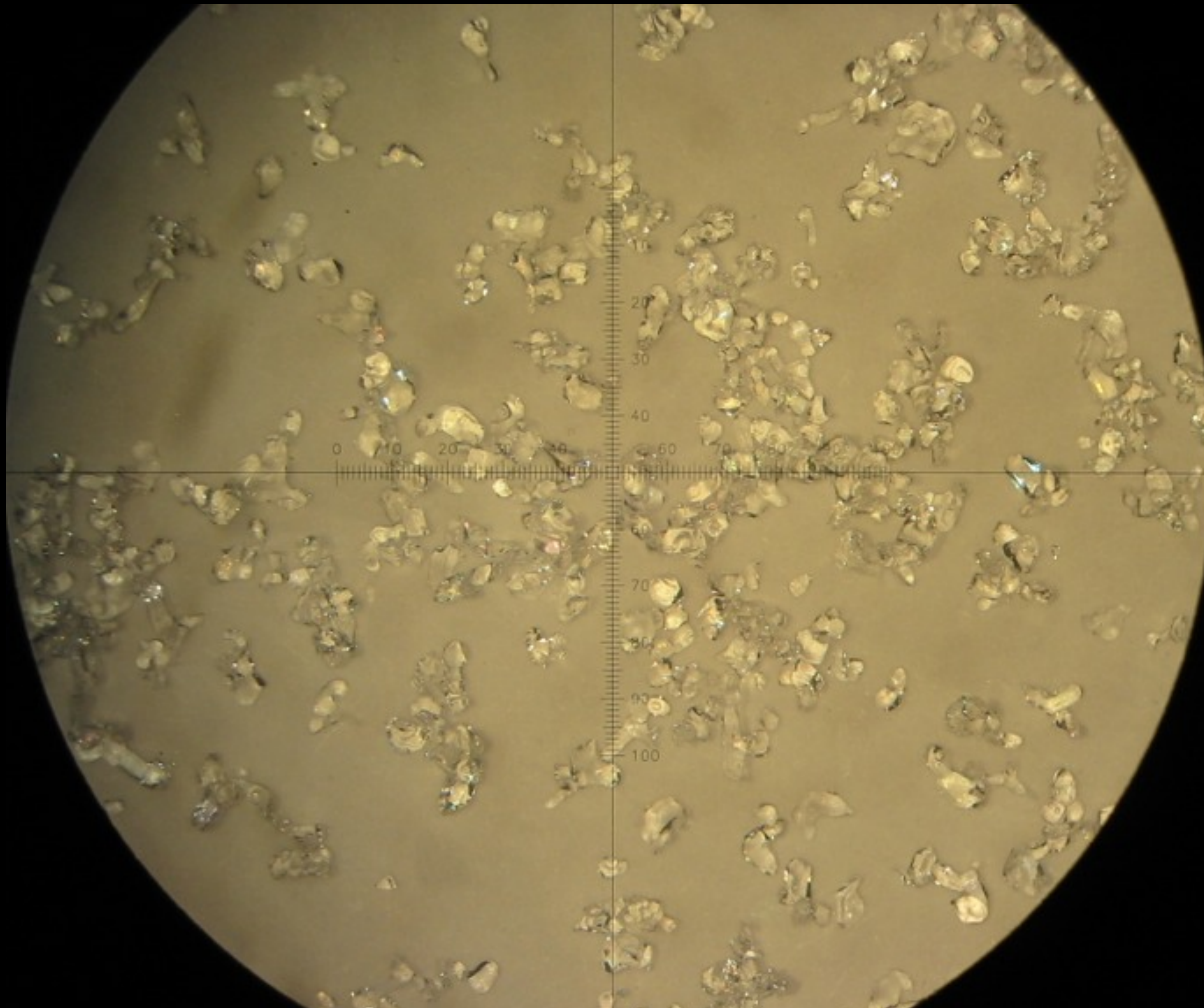
Different results

Bayesian inversion method:

Look-up tables: FAST

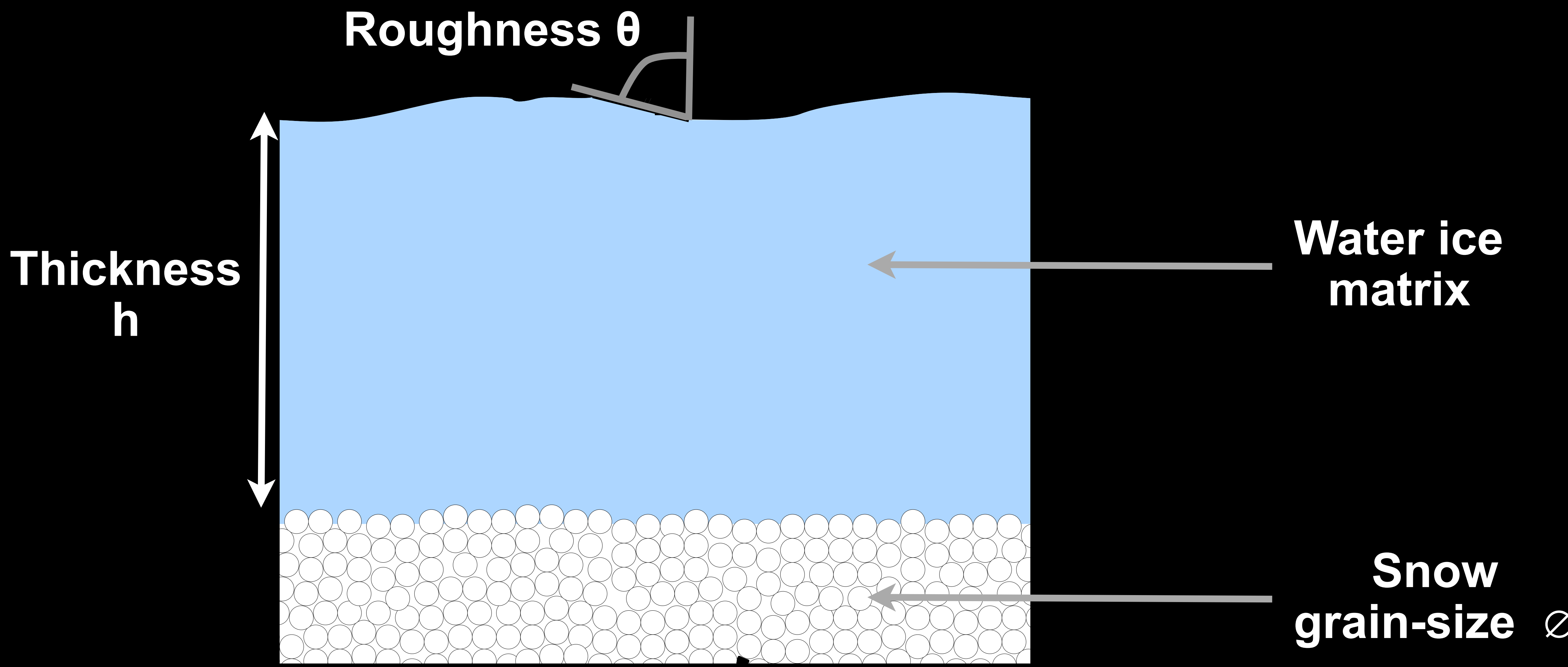
Bayesian: STATISTICS
- **on the data**
- **on the results**

Experimental validations:



Measure at IPAG, Grenoble, France (Brissaud *et al.*, 2004)

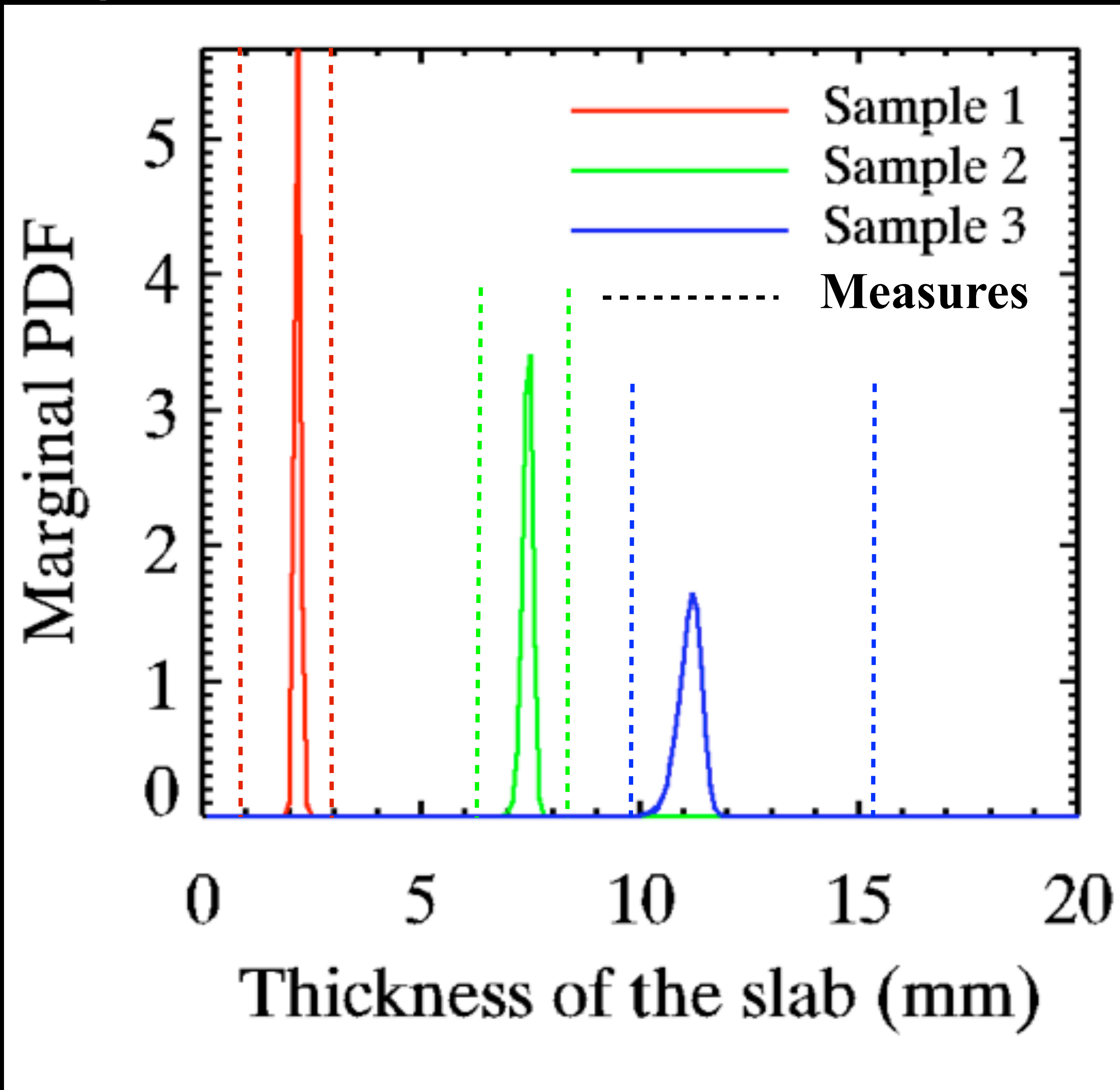
Experimental validations:



Laboratory optical constants H_2O from Schmitt et al. ,1998

Direct model from Andrieu et al., 2015

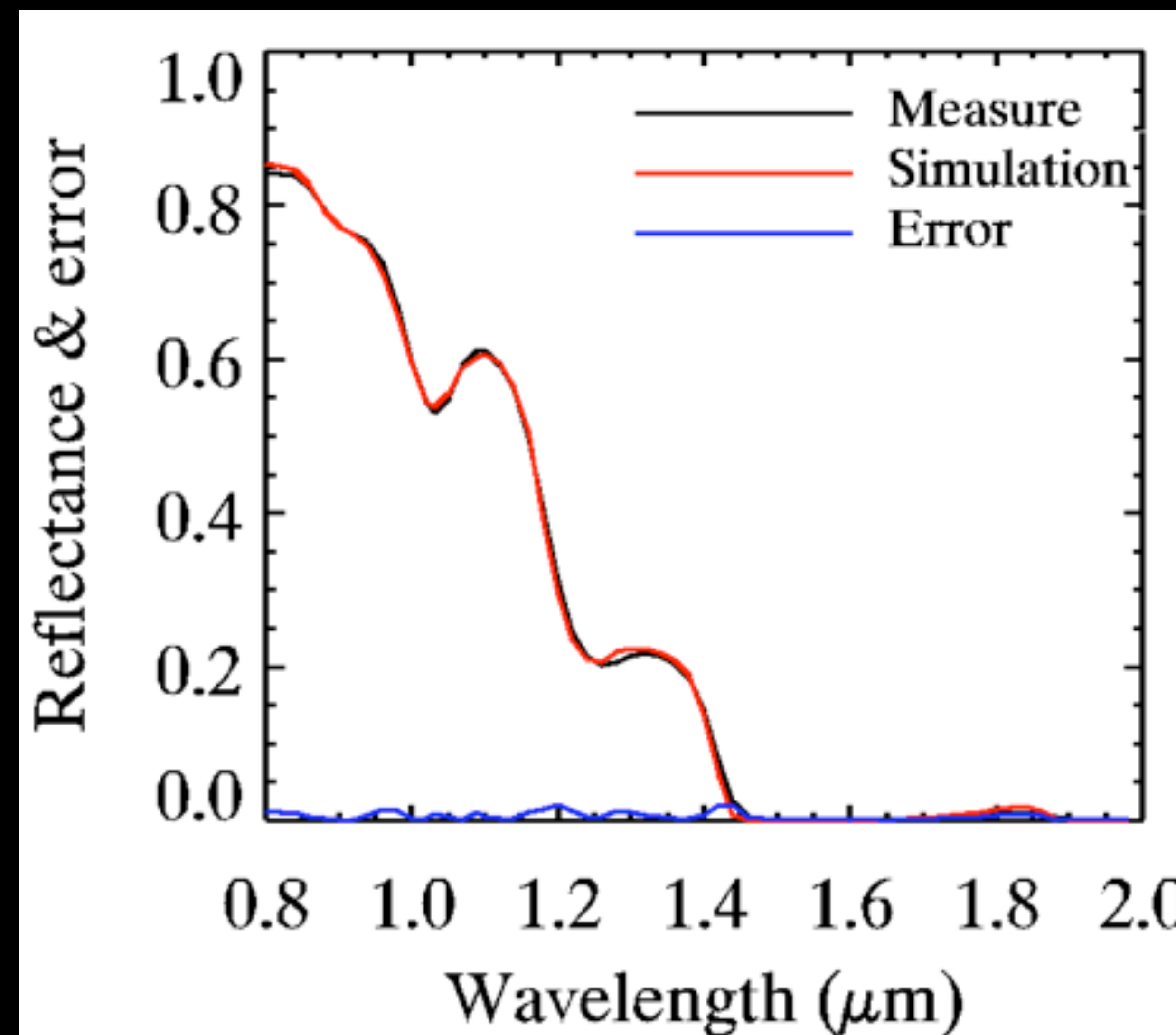
1) Thickness of the slab: a posteriori PDF



Laboratory optical constants H_2O (Schmitt *et al.* ,1998)

Thickness of the slab :

Measured :	Retreived :
Sample 1 : 1.42 ± 0.54 mm	Sample 1 : 2.1 ± 0.14 mm
Sample 2 : 7.45 ± 0.84 mm	Sample 2 : 7.4 ± 0.28 mm
Sample 3 : 12.51 ± 2.7 mm	Sample 3 : 11.2 ± 0.5 mm



Andrieu et al., Retrieving the characteristics of slab ice covering snow by remote sensing, The Cryosphere Discuss., 9, 5137-5169, 2015

Numerical validations : inversion of the LUT

For each spectrum :

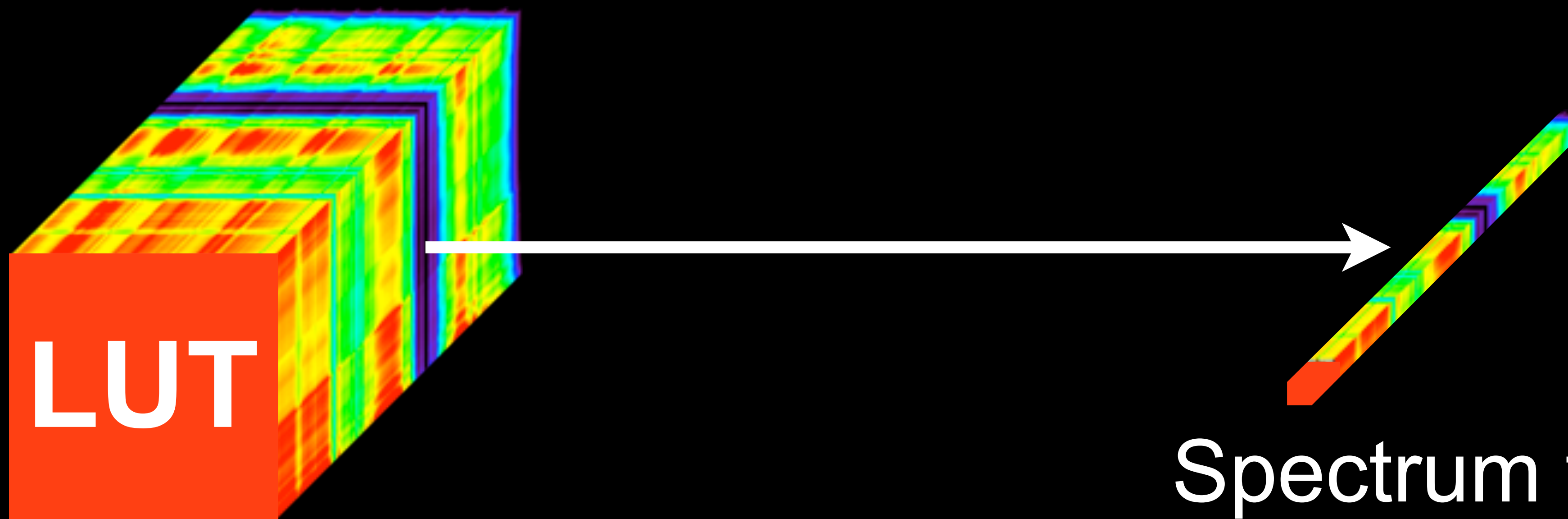
- application of a gaussian noise
- inversion

x1000

- stack of the 1000 *a posteriori* PDF

→ **Distribution of the PDF = evaluation of the inversion**

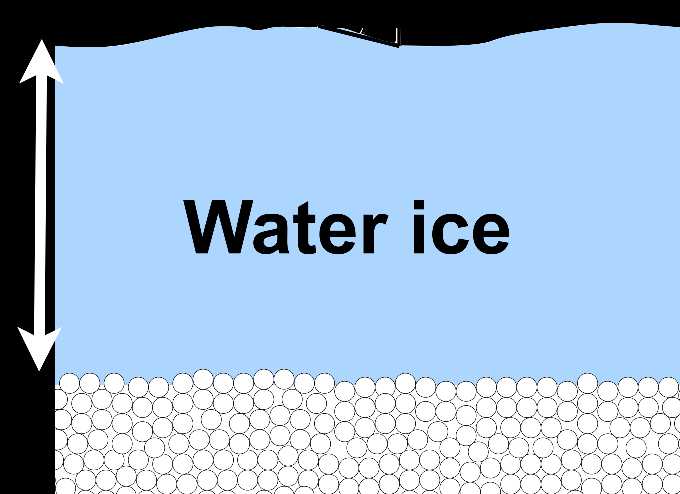
Numerical validations : inversion of the LUT



Spectral database
(**model** LUT)

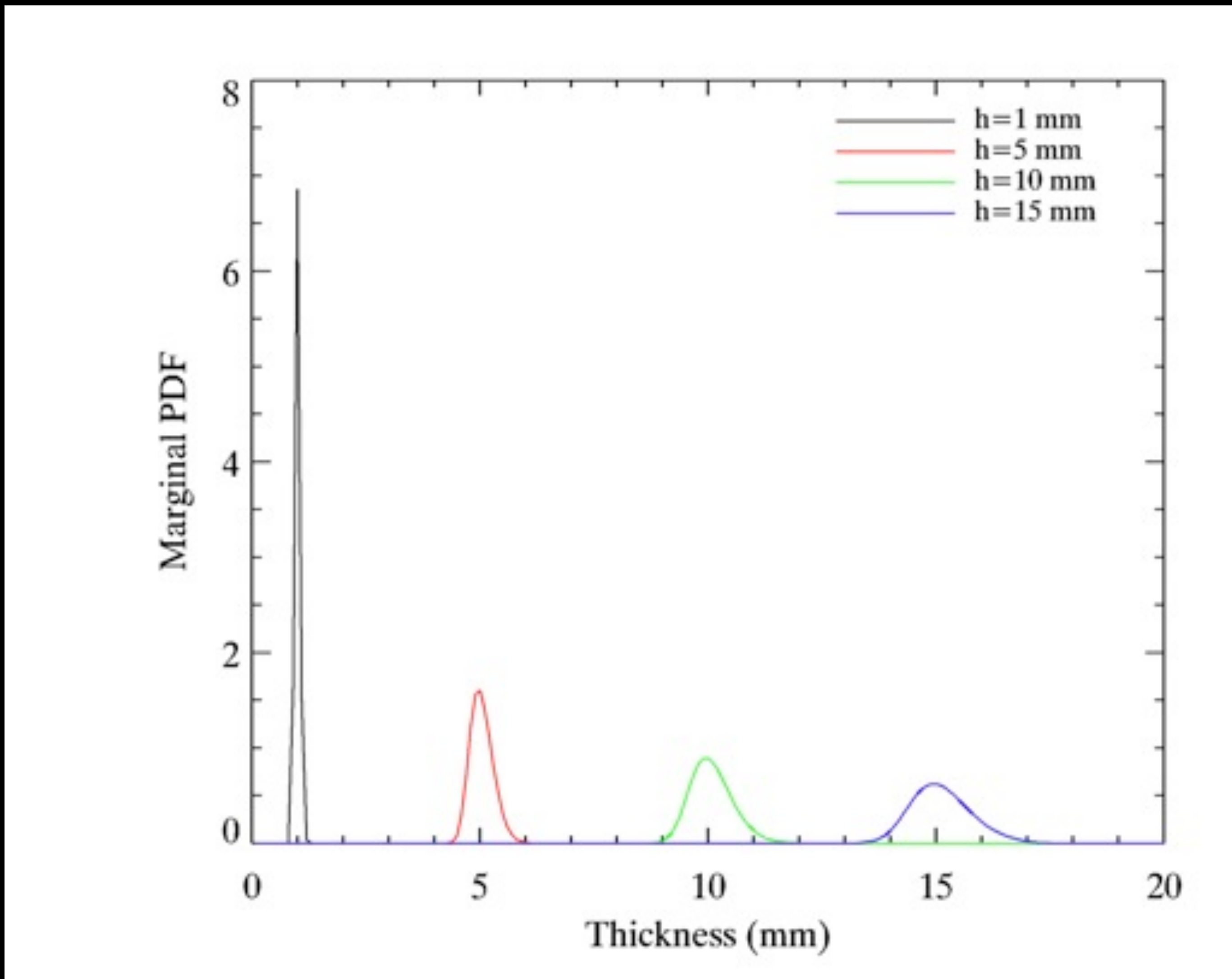
Spectrum from the
LUT

Thickness
h
0mm<h<20mm
step=0.1mm

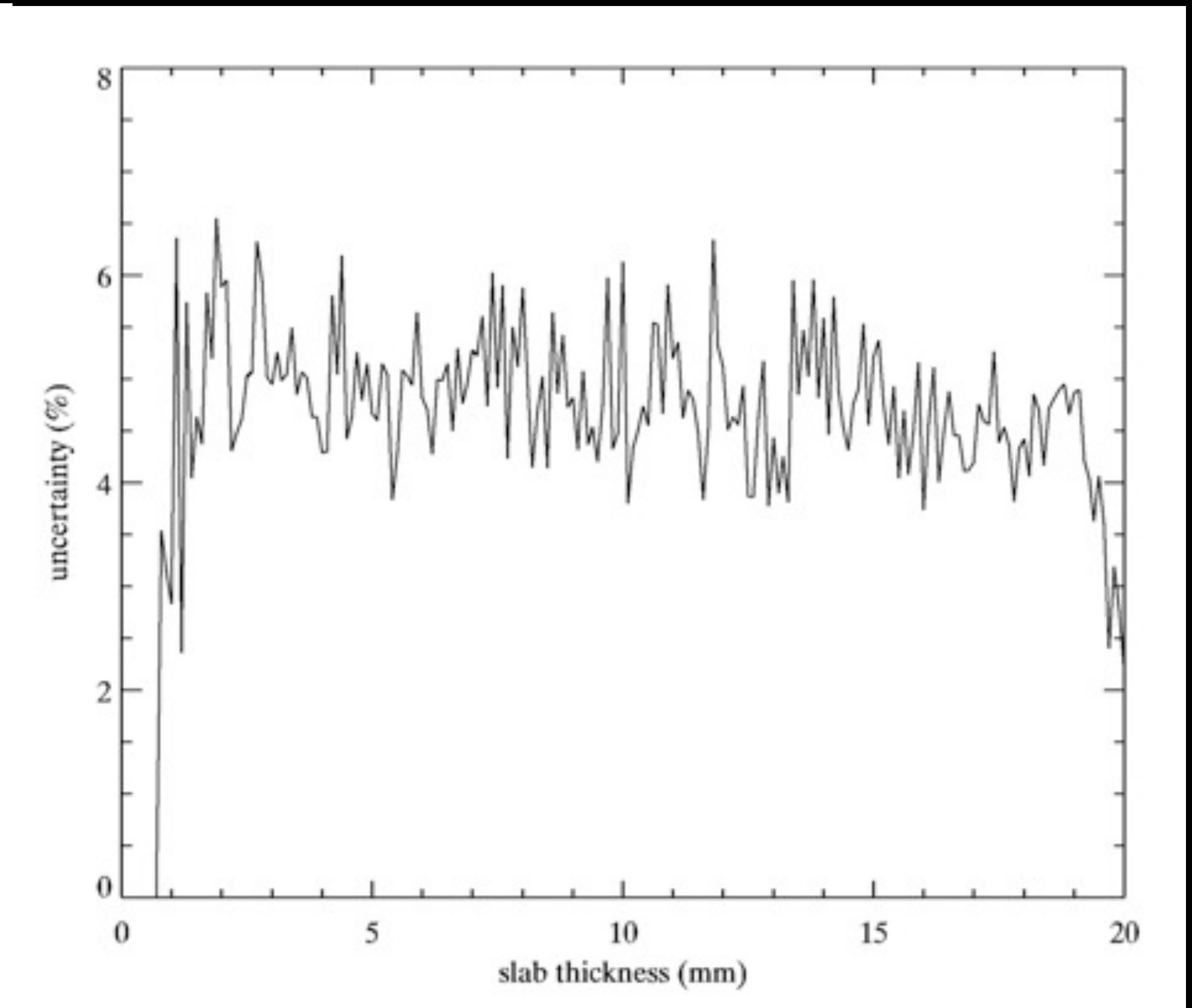


Snow
grain-size ∅
1µm< ∅<500µm
step=10µm

Numerical validations : *a posteriori* uncertainties



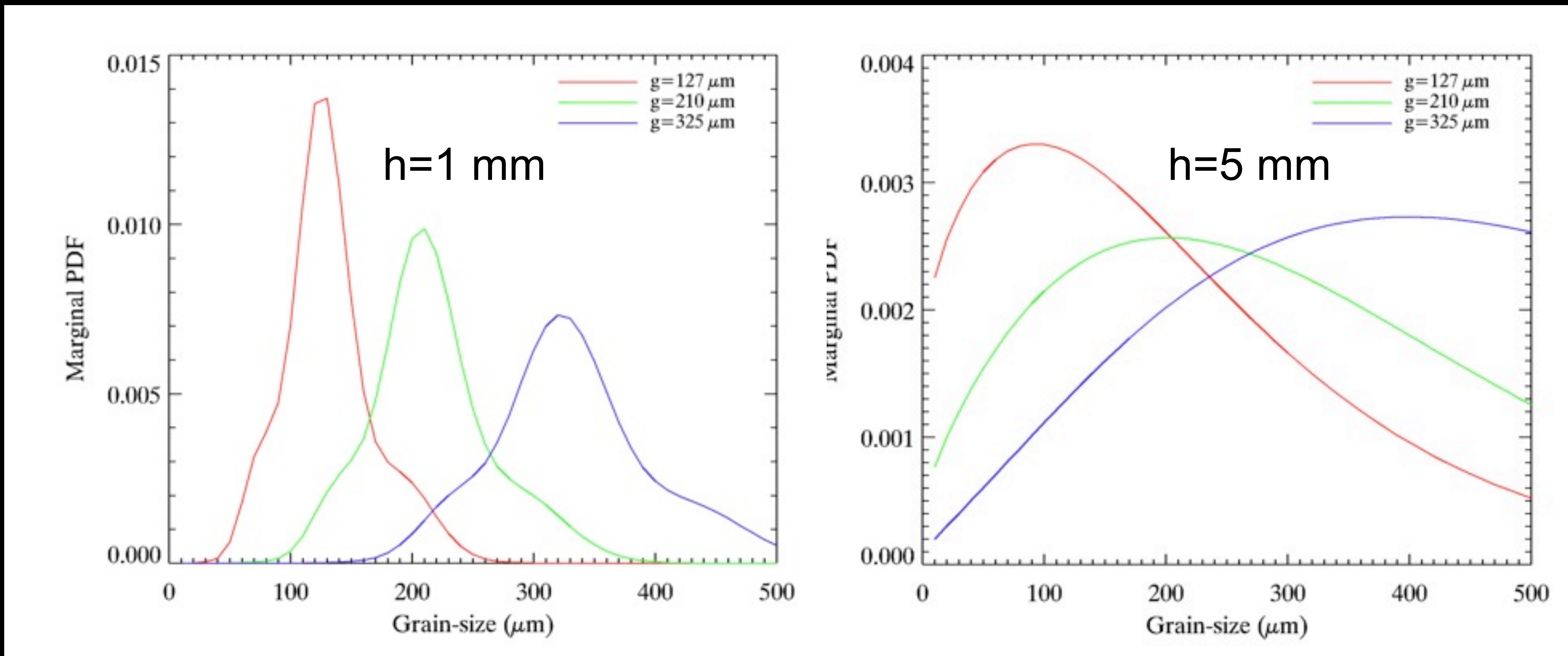
Stacks of 1000 *a posteriori* PDF for 4 different inversions



Evolution of *a posteriori* uncertainty (2σ) with the thickness

$\sigma_{noise} = 2\ \%$

Numerical validations : *a posteriori* uncertainties

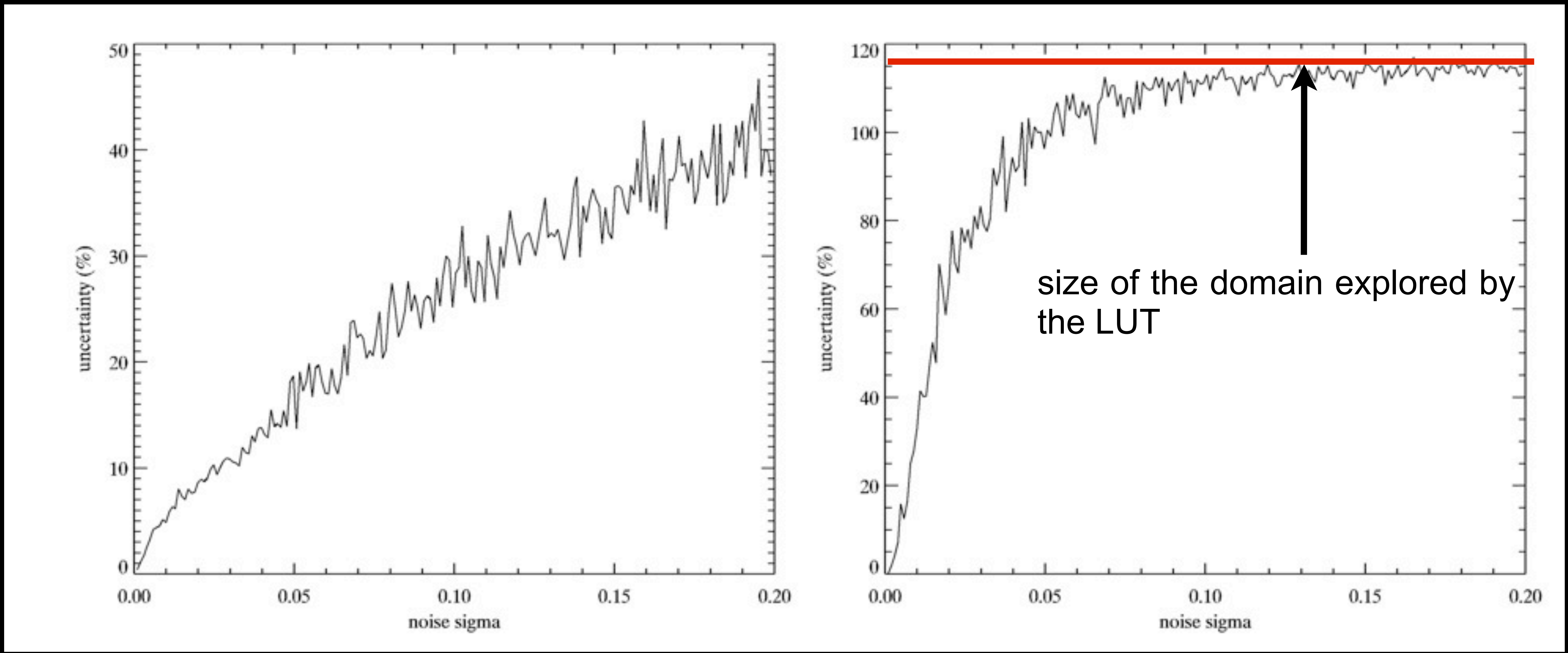


Stacks of 1000 *a posteriori* PDF for 3 different inversions

Stacks of 1000 *a posteriori* PDF for 3 different inversions

$\sigma_{noise} = 2\ \%$

Numerical validations : *a posteriori* uncertainties



Evolution of *a posteriori* uncertainty (2σ) on the thickness with the noise

Evolution of *a posteriori* uncertainty (2σ) on the grain-size with the noise

«good scenario»

«bad scenario»

Conclusions:

Bayesian inversion method:

Look-up tables: FAST

Bayesian: STATISTICS
- on the data
- on the results

Applicability:

Any hyperspectral dataset