Collaborative Total Variation for Hyperspectral Pansharpening

Mauro Dalla Mura
mauro.dalla-mura@gipsa-lab.grenoble-inp.fr

GIPSA-Lab, Grenoble Institute of Technology, France

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Joint work with. . .

- **Addesso Paolo**, University of Salerno, Italy
- **Condat Laurent**, GIPSA-lab, CNRS, France
- **Restaino Rocco**, University of Salerno, Italy
- **Vivone Gemine**, University of Salerno, Italy
- **Picone Daniele**, University of Salerno, Italy
- **Chanussot Jocelyn**, GIPSA-lab, Grenoble Institute of Technology, France

Outline

What is pansharpening?

Pansharpening: Sharpening (i.e., enhancing) a multi-/hyper-spectral image with a panchromatic one.

Panchromatic (PAN) image

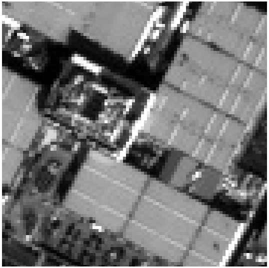
Multispectral (MS) image

What is pansharpening?

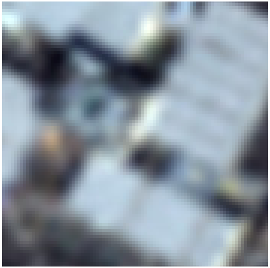
Pansharpened (PS) image

Issues

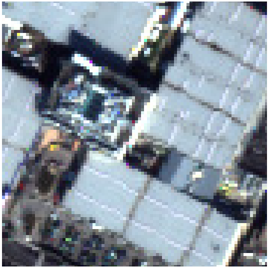
- Spatial details that are present in the PAN appear blurred in the MS channels → due to the different spatial resolution
- Details appear with variable intensity in the different spectral channels according to their spectral signature
- Retrieving the specific spectral contributions is difficult due to the absent spectral information in the PAN



PAN



MS



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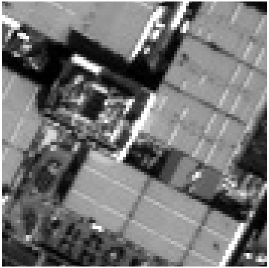

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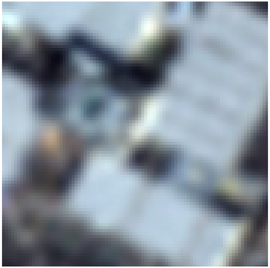

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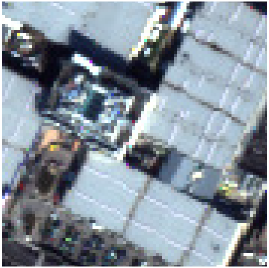

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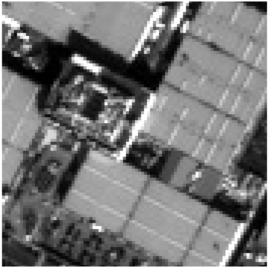

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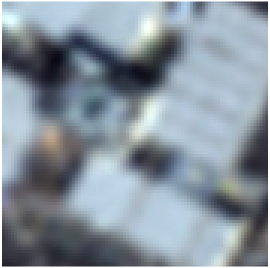

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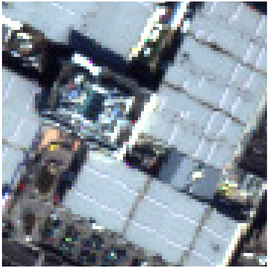

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The pansharpening model

- **Classical approach**: extract the spatial details from the PAN that are not resolved in the MS and inject them (opportunely modulated) into the MS
- **Notation**

$\widehat{\mathbf{MS}}$	result of pansharpening
$\widetilde{\mathbf{MS}}$	MS image upscaled to the size of the PAN
$\mathbf{P}$	the PAN image
$\mathbf{P}_D$	spatial details of the PAN

- **Pansharpening**

$$\widehat{\mathbf{MS}}_k = \widetilde{\mathbf{MS}}_k + g_k \mathbf{P}_D,$$

  - k denotes the k-th spectral channel over N bands
  - $\mathbf{g} = [g_1, \ldots, g_k, \ldots, g_N]$ are the injections gains

Two typical approaches are employed according to the technique used for estimating $\mathbf{P}_D$:

1. Component Substitution (CS) → the details are estimated considering the MS
2. Multi-Resolution Analysis (MRA) → the details are estimated by filtering the PAN

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- Detail extraction

$$\mathbf{P}_D = \mathbf{P} - \mathbf{I}_L$$

- $\mathbf{I}_L$ a monochromatic image obtained by the weighted linear composition of the MS upsampled bands obtained as

$$\mathbf{I}_L = \sum_{k=1}^{N} w_k \widetilde{\mathbf{MS}}_k.$$

- Equivalent implementation of CS (under linear hypothesis):
  1. perform a spectral transformation of the MS into another feature space in which the spatial and spectral contributions are more separated
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MultiResolution Analysis

- MRA is based on the extraction of the spatial details present in the PAN (and not fully resolved in the multispectral one) and their subsequent injection to the MS bands

$$\widehat{\mathbf{MS}}_k = \widetilde{\mathbf{MS}}_k + g_k(\mathbf{P} - \mathbf{P}_L)$$

- Thus for MRA techniques the details are extracted as

$$\mathbf{P}_D = \mathbf{P} - \mathbf{P}_L,$$

- $\mathbf{P}_L$ a low pass version of the PAN image obtained by spatially filtering $\mathbf{P}$ (e.g., $\mathbf{P}_L = \mathbf{P} * h$, with h a mask implementing a low-pass filter and $*$ the product of convolution)
- The spatial details can be extracted by several approaches as using an average filter or multiresolution decompositions of the image based on Laplacian pyramids, or wavelet/contourlet operators
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Injecting $\mathbf{P}_D$ in the MS

- For both CS and MRA families, $\mathbf{P}_D$ has to be injected into the interpolated MS bands
- The injection is done by weighting $\mathbf{P}_D$ by the coefficients g_k
- g_k are in general different for each band
- "global" pansharpening techniques consider the same g_k for all the pixels in each channel
- "local" approaches allow g_k to vary locally in the spatial domain of the image

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Hyperspectral Pansharpening [Simoes16]

Notation

- HS image $\mathbf{H} \in \mathbb{R}^{C \times N_{HS}}$, with C bands and N_{HS} pixels
- PAN image $\mathbf{P} \in \mathbb{R}^{1 \times N_{PAN}}$, $N_{PAN} = \rho^2 N_{HS}$, where $\rho > 1$ is the *resolution ratio*
- Goal: high spatial and spectral resolution image $\mathbf{Z} \in \mathbb{R}^{C \times N_{PAN}}$

HS model

$$\mathbf{H} = \mathbf{ZBM} + \mathbf{N}_h$$

- $\mathbf{B} \in \mathbb{R}^{N_{PAN} \times N_{PAN}}$ blur
- $\mathbf{M} \in \mathbb{R}^{N_{PAN} \times N_{HS}}$ subsampling
- $\mathbf{N}_h \in \mathbb{R}^{C \times N_{HS}}$ (i.i.d.) zero-mean Gaussian noise with variance σ^2_{HS}

PAN model

$$\mathbf{P} = \mathbf{RZ} + \mathbf{N}_p$$

- $\mathbf{R} \in \mathbb{R}^{1 \times C}$ is related to the Relative Spectral Response of PAN
- $\mathbf{N}_p \in \mathbb{R}^{1 \times N_{PAN}}$ (i.i.d.) zero-mean Gaussian noise with variance σ^2_{PAN}

- $\mathbf{Z}$ is a hyperspectral image $\mathbf{H} \in \mathbb{R}^{C \times N_{PAN}}$, with C bands and N_{PAN} pixels (organized lexicographically order)
- $\mathbf{Z}$ lives in a subspace of dimensionality lower than C
- $\Rightarrow$ Factorize $\mathbf{Z}$ as $\mathbf{Z} = \mathbf{EX}$ with
  - $\mathbf{E}$ is the set of basis (with cardinality $L \leq C$) spanning the subspace of $\mathbf{Z}$
  - $\mathbf{X}$ are the representation coefficients
- Factorization
  - representation on a subspace e.g., Singular Value Decomposition
  - representation on a simplex (spectral unmixing) (e.g., Vertex Component Analysis [Nascimento05] + FCLSU)
  - ...

With $\mathbf{Z} = \mathbf{EX}$

$$\underset{\mathbf{X}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{H} - \mathbf{EXBM}\|_F^2 + \frac{\lambda_m}{2} \|\mathbf{P} - \mathbf{REX}\|_F^2 + \lambda_\varphi \varphi(\mathbf{X})$$

where $\|\cdot\|_F$ is the Frobenius norm and $\lambda_m = 1$

Total Variation

Total variation [Rudin92]

$$TV(X) = \int_{\Omega} |\nabla X(x)| dx$$

for a given a scalar function $X : (\Omega \subseteq \mathbb{R}^n) \to \mathbb{R}$
Used as a regularizer in many optimization problems in imaging.
For example:

$$\underset{X}{\text{minimize}} \quad ||X||_{TV} + \frac{\lambda}{2}||X - X^0||_2^2$$

Total variation for color images

For a multivariate image $\mathbf{X} : (\Omega \subseteq \mathbb{R}^n) \to \mathbb{R}^m$ [Blomgren98] proposed

$$TV_{n,m}(\mathbf{X}) = \sqrt{\sum_{i=1}^{m} [TV_{n,1}(\mathbf{X}_i)]^2}$$

However other alternative definitions are possible.

Collaborative Total Variation (CTV)

Literature	Continuous Formulation	Collaborative TV
[1]	$\sum_{k=1}^{C}\int_\Omega \sqrt{(\partial_x u_k(x))^2+(\partial_y u_k(x))^2}\,dx$	$\ell^{2,1,1}(der,pix,col)$
Anisotropic variant	$\sum_{k=1}^{C}\int_\Omega \left(\lvert\partial_x u_k(x)\rvert+\lvert\partial_y u_k(x)\rvert\right)dx$	$\ell^{1,1,1}(der,pix,col)$
[4]	$\sqrt{\sum_{k=1}^{C}\left(\int_\Omega \sqrt{(\partial_x u_k(x))^2+(\partial_y u_k(x))^2}\,dx\right)^2}$	$\ell^{2,1,2}(der,pix,col)$
Anisotropic variant	$\sqrt{\sum_{k=1}^{C}\left(\int_\Omega \left(\lvert\partial_x u_k(x)\rvert+\lvert\partial_y u_k(x)\rvert\right)dx\right)^2}$	$\ell^{1,1,2}(der,pix,col)$
[6] [47]	$\int_\Omega \sqrt{\sum_{k=1}^{C}\left((\partial_x u_k(x))^2+(\partial_y u_k(x))^2\right)}\,dx$	$\ell^{2,2,1}(der,col,pix)$
Anisotropic variants	$\int_\Omega \left(\sqrt{\sum_{k=1}^{C}(\partial_x u_k(x))^2}+\sqrt{\sum_{k=1}^{C}(\partial_y u_k(x))^2}\right)dx$	$\ell^{2,1,1}(col,der,pix)$
	$\int_\Omega \sqrt{\sum_{k=1}^{C}\left(\lvert\partial_x u_k(x)\rvert+\lvert\partial_y u_k(x)\rvert\right)^2}\,dx$	$\ell^{1,2,1}(der,col,pix)$
Strong coupling	$\int_\Omega \left(\max_{1\leq k\leq C}\lvert\partial_x u_k(x)\rvert+\max_{1\leq k\leq C}\lvert\partial_y u_k(x)\rvert\right)dx$	$\ell^{\infty,1,1}(col,der,pix)$
	$\int_\Omega \max_{1\leq k\leq C}\left(\lvert\partial_x u_k(x)\rvert+\lvert\partial_y u_k(x)\rvert\right)dx$	$\ell^{1,\infty,1}(der,col,pix)$
Isotropic variants	$\int_\Omega \sqrt{\left(\max_{1\leq k\leq C}\lvert\partial_x u_k(x)\rvert\right)^2+\left(\max_{1\leq k\leq C}\lvert\partial_y u_k(x)\rvert\right)^2}\,dx$	$\ell^{\infty,2,1}(col,der,pix)$
	$\int_\Omega \max_{1\leq k\leq C}\sqrt{(\partial_x u(x))^2+(\partial_y u(x))^2}\,dx$	$\ell^{2,\infty 1}(der,col,pix)$
Supremum variant	$\int_\Omega \left(\max\left\{\max_{1\leq k\leq C}\lvert\partial_x u_k(x)\rvert, \max_{1\leq k\leq C}\lvert\partial_y u_k(x)\rvert\right\}\right)dx$	$\ell^{\infty,\infty,1}(col,der,pix)$
[34] [47]	$\int_\Omega \sum_{i=1}^{r}\lvert\sigma_i(\nabla u(x))\rvert\,dx$	$(S^1(col,der),\ell^1(pix))$
Frobenius norm	$\int_\Omega \sqrt{\sum_{i=1}^{r}(\sigma_i(\nabla u(x)))^2}\,dx$	$(S^2(col,der),\ell^1(pix))$
[23] [47]	$\int_\Omega \max_{1\leq i\leq r}\sigma_i(\nabla u(x))\,dx$	$(S^\infty(col,der),\ell^1(pix))$

Table 1: Overview of local vectorial TV approaches and the way they fit in our framework.

[Duran15]

Collaborative Total Variation

For $\mathbf{A} = \{\partial_x \mathbf{f}, \partial_y \mathbf{f}\} \in \mathbb{R}^{N\times L\times M}$ two family of norms are considered

- $||\cdot||_{p,q,r} : \ell^{p,q,r}(der, bands, pix)$
- $(\mathbb{S}^p, \ell^q)(der, bands, pix)$

$$||\mathbf{A}||_{p,q,r} = \left(\sum_{i=1}^{N} \left(\sum_{j=1}^{L} \left(\sum_{k=1}^{M} |\mathbf{A}_{i,j,k}|^p \right)^{q/p} \right)^{r/q} \right)^{1/r}$$

$$(\mathbb{S}^p, \ell^q)(\mathbf{A}) = \left(\sum_{i=1}^{N} \left\| \begin{matrix} \mathbf{A}_{i,1,1} & \cdots & \mathbf{A}_{i,1,M} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{i,L,1} & \cdots & \mathbf{A}_{i,L,M} \end{matrix} \right\|_{\mathbb{S}^p}^{q} \right)^{1/q}$$

Denoising and Inpainting

- Inpainting: missing data in the *inpainting domain* $\mathcal{I} \subseteq \Omega \subset \mathbb{R}^N$
- Denoising: $\mathcal{I} = \varnothing$

Convex formulation

$$\underset{\mathbf{Z}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{H} - \mathbf{Z}\|^2_{F(\overline{\mathcal{I}})} + \lambda_\varphi \varphi(\mathbf{Z}),$$

with

- $\|\cdot\|_{F(\overline{\mathcal{I}})}$ is the Frobenius norm on the complement of the inpainting domain $\mathcal{I}$
- $\varphi(\mathbf{Z})$ is a regularization term with a coefficient λ_φ

Factorize $\mathbf{Z}$ as $\mathbf{Z} = \mathbf{EX}$ with $\mathbf{E}$ is the set of L basis and $\mathbf{X}$ are the representation coefficients

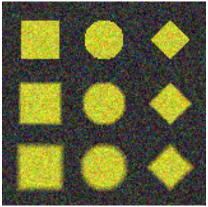
$$\underset{\mathbf{X}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{H} - \mathbf{EX}\|^2_{F(\overline{\mathcal{I}})} + \lambda_\varphi \varphi(\mathbf{X}).$$

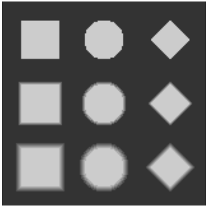
Denoising on a synthetic image

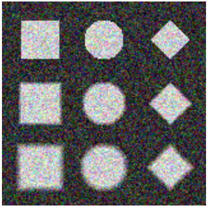
4 bands image (160 × 160 pixels)

- Different shapes and edge transitions
- Case 1, shapes present in bands 2 and 3
- Case 2, shapes present in all bands
- SNR = 10 dB

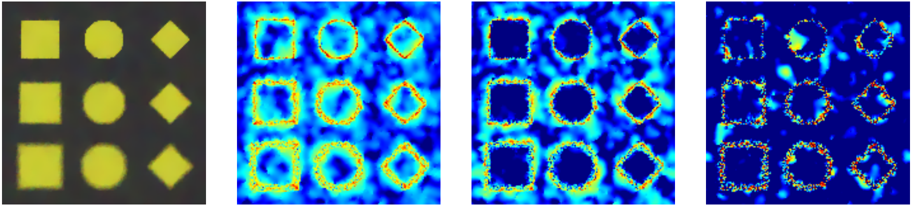

Case 1 Reference


Case 1 Noisy


Case 2 Reference


Case 2 Noisy

$\ell^{2,2,1}(dbx)$



$\ell^{1,1,1}(bdx)$

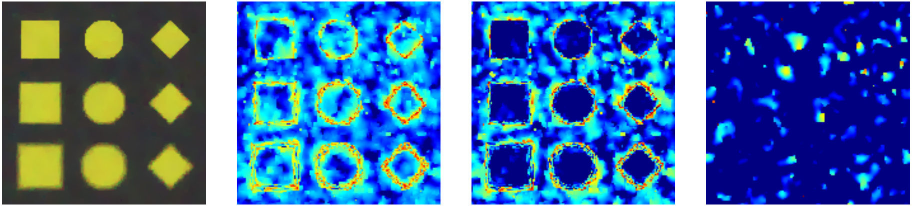
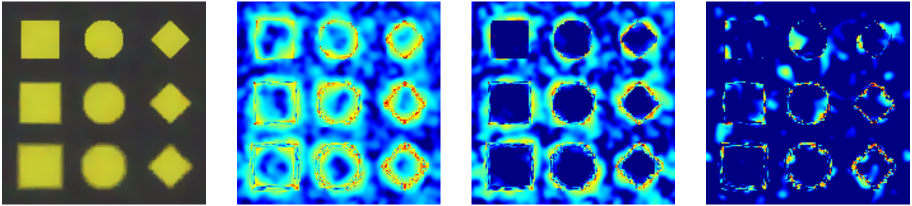


Image | MSE | SAM | MSE (Band 1)

Denoising - Results Case 1

$\ell^{2,1,1}(bdx)$



$\ell^{\infty,1,1}(bdx)$

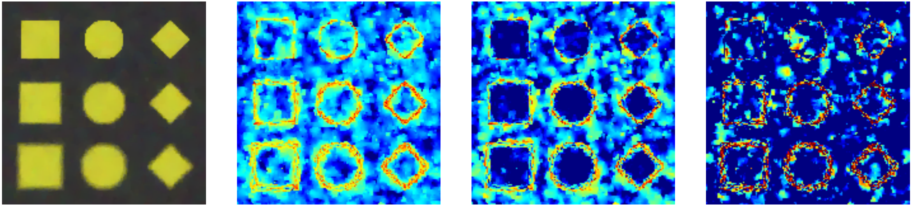
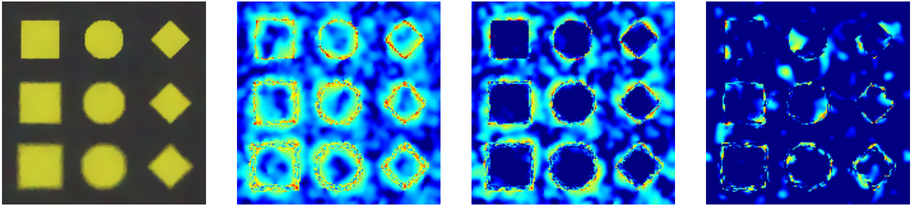


Image | MSE | SAM | MSE (Band 1)

Denoising - Results Case 1

$(\mathbb{S}^1(bd), \ell^1(x))$



$(\mathbb{S}^\infty(bd), \ell^1(x))$

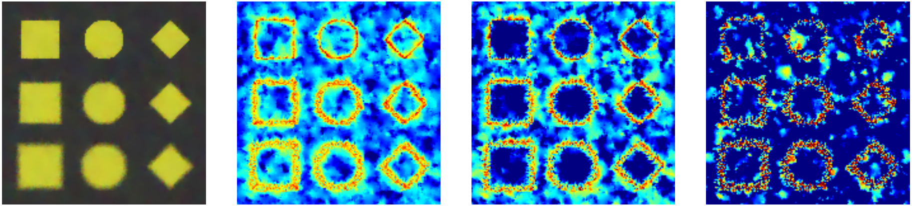


Image | MSE | SAM | MSE (Band 1)

Denoising - Results Case 2

$\ell^{2,2,1}(dbx)$

$\ell^{1,1,1}(bdx)$

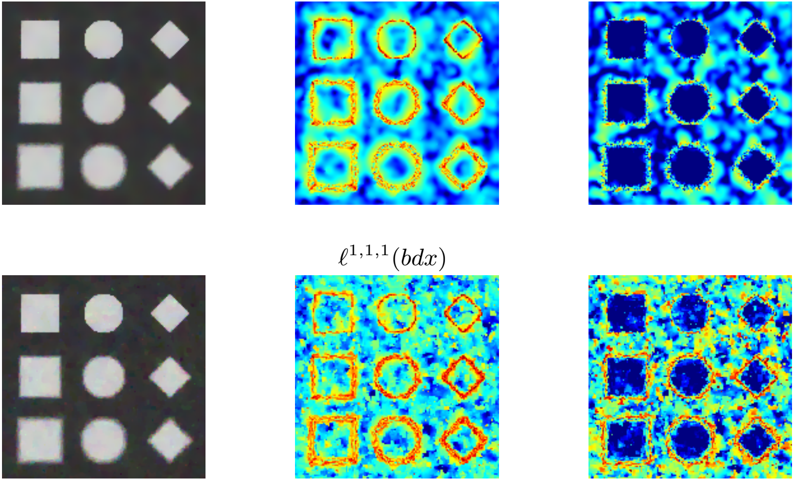
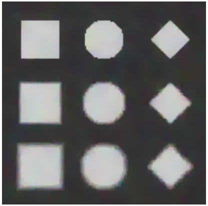
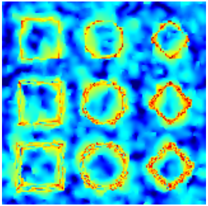
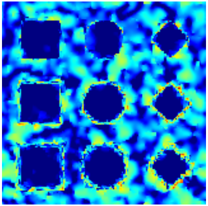


Image MSE SAM

Denoising - Results Case 2

$\ell^{2,1,1}(bdx)$

$\ell^{\infty,1,1}(bdx)$

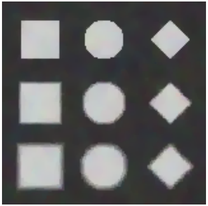
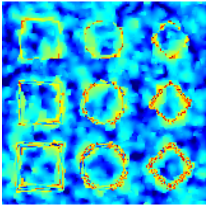
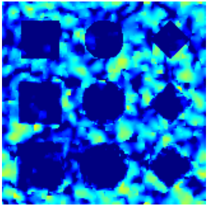
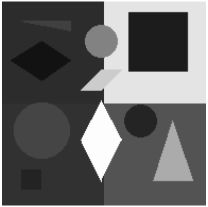
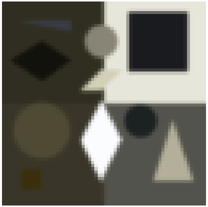
  

Image MSE SAM

Synthetic dataset


Reference

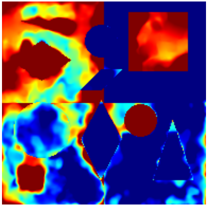

PAN


HS

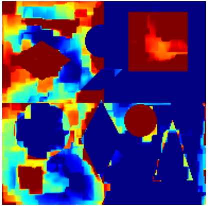

$\ell^{2,2,1}(dbx)$

Results

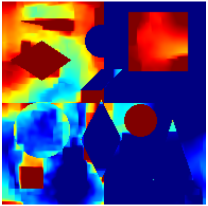
Synthetic dataset ($SNR_{PAN} = 40$ dB, $SNR_{HS} = 30$ dB)



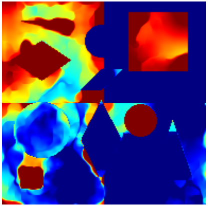
SAM $\ell^{2,2,1}(dbx)$



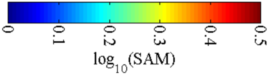
$\ell^{1,1,1}(bdx)$



$\ell^{2,1,1}(bdx)$



$(\mathbb{S}^1(bd), \ell^1(x))$



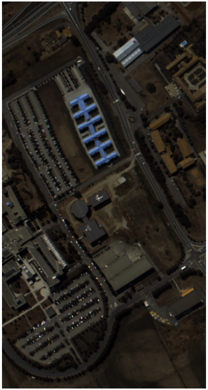
Reference

Synthetic dataset: performance obtained by the tested CTV norm averaged on 100 Monte Carlo trials. λ_φ is the optimal value of the TV term weight

Norm	$SNR_{PAN} = 40$ dB, $SNR_{HS} = 30$ dB					$SNR_{PAN} = 20$ dB, $SNR_{HS} = 20$ dB					**T** [s]
	λ_φ	**ERGAS**	**SAM**	**UIQI**	**SCC**	λ_φ	**ERGAS**	**SAM**	**UIQI**	**SCC**	
$\ell^{2,2,1}(dbx)$	0.05	0.9919	1.8793	0.9334	0.9558	0.67	6.2949	11.438	0.7648	0.8799	39.9
$\ell^{1,1,1}(bdx)$	0.02	0.9968	2.0026	0.9336	0.9560	0.37	6.4758	<u>11.111</u>	0.7635	0.8483	45.8
$\ell^{2,1,1}(bdx)$	0.05	**0.9095**	**1.7430**	**0.9359**	**0.9567**	0.6	**6.0433**	11.142	<u>0.7676</u>	<u>0.8814</u>	28.9
$\ell^{\infty,1,1}(bdx)$	0.1	0.9832	1.9108	0.9347	0.9558	1.44	6.2674	11.284	0.7645	0.8669	130.3
$\ell^{\infty,\infty,1}(bdx)$	0.15	1.2594	2.0283	0.9290	0.9540	2.33	7.3584	11.432	0.7503	0.8160	121.5
$\ell^{2,\infty,1}(dbx)$	0.1	1.1538	2.2805	0.9285	0.9544	1.89	6.8250	11.446	0.7566	0.8438	92.4
$(\mathbb{S}^1(bd), \ell^1(x))$	0.05	<u>0.9330</u>	<u>1.7455</u>	<u>0.9353</u>	**0.9567**	0.67	<u>6.0791</u>	**11.041**	**0.7685**	**0.8821**	45.5
$(\mathbb{S}^\infty(bd), \ell^1(x))$	0.075	1.1306	2.0097	0.9280	0.9550	0.83	7.1989	11.964	0.7495	0.8398	80.1

Results

Pavia University dataset



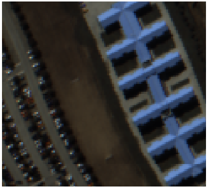
Reference image

PAN

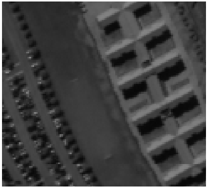
HS

Fused (CTV $\ell^{2,2,1}(dbx)$)

Pavia University dataset



Reference image



PAN

HS

Fused (CTV $\ell^{2,2,1}(dbx)$)

Pavia University dataset: performance obtained by some CTV, CS and MRA algorithms, averaged on 100 Monte Carlo trials. λ_φ is the optimal value of the TV

Algorithm	$SNR_{PAN} = 40$ dB, $SNR_{HS} = 30$ dB					$SNR_{PAN} = 20$ dB, $SNR_{HS} = 20$ dB				
	λ_φ	**ERGAS**	**SAM**	**UIQI**	**SCC**	λ_φ	**ERGAS**	**SAM**	**UIQI**	**SCC**
EXP	-	7.3828	5.2903	0.7686	0.5517	-	7.4267	5.7044	0.7623	0.5533
HPF	-	5.8474	7.0084	0.8792	0.7261	-	6.8810	8.0241	0.8105	0.7345
ATWT	-	5.9789	8.0134	0.8789	0.7404	-	6.9640	8.8917	0.8154	0.7469
GS	-	5.3330	6.3522	0.8839	0.7411	-	6.1975	7.0914	0.8276	0.7433
GSA	-	6.0100	9.2121	0.8798	0.7409	-	7.4846	10.501	0.8056	0.7460
PCA	-	7.4006	9.3170	0.7843	0.6854	-	8.0904	9.9096	0.7387	0.6899
CTV: $\ell^{2,2,1}(dbx)$	0.002	3.8160	4.8204	0.9411	**0.7804**	0.02	4.2458	5.0985	0.9168	0.7494
CTV: $\ell^{1,1,1}(bdx)$	0.002	3.8431	4.8398	0.9387	0.7777	0.005	4.2891	**5.0138**	0.9147	**0.7679**
CTV: $\ell^{2,1,1}(bdx)$	0.002	3.9325	4.9662	0.9370	0.7763	0.01	4.3937	5.2126	0.9116	0.7600
CTV: $(\mathbb{S}^1(bd), \ell^1(x))$	0.002	**3.7809**	**4.7396**	**0.9421**	0.7801	0.02	**4.2097**	5.0285	**0.9179**	0.7436

Conclusions

- Pansharpening allows one to enhance the spatial resolution of an multi-/hyper-spectral image
- We proposed a technique based on CTV
- Different effects are obtained according to the coupling that is enforced on the spectral channels
- Relevant to consider different formulations for HSI

Next steps

- Weight differently the bands $\underset{\mathbf{Z}}{\text{minimize}} \quad \frac{1}{2} \|(\mathbf{H} - \mathbf{Z})\mathbf{W}\|^2_{F(\overline{\mathcal{I}})} + \lambda_\varphi \varphi(\mathbf{Z})$
- CTV for a discrete formulation of TV [Condat17]
- Inpainting in HSI videos (CTV: D_h, D_v, D_t)

Thanks for your attention!